Forum for matematiske perler

Tid:	Fredag 7.2.03, kl. 12.15 – 13.00
Sted:	Rom 1236, Sentralbygg II, Gløshaugen
Foredragsholder:	Peter Lindqvist
Tittel:	"Old and New about Odd Zeta Values and Some Other Remarks".

(Som vanlig blir det servert kaffe og bløtkake som kan nytes under foredraget.)

Sammendrag: To the majority of the mathematicians the theory of the Zeta Function of Riemann is like a ferocious jungle of elaborate formulas and complicated estimates. Yet, it is a theme of central importance, because the Zeta Function is related to the distribution of the prime numbers. We shall put "the ferocious jungle" aside and concentrate on an isolated problem, where some progress has been recently reported.

In 1689 Jacob Bernoulli wrote the following about the sum

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} + \cdots$$

in *Tractatus de seriebus infinitis*: Si quelqu'un détermine et nous communique ce qui a jusqu'ici échappé à tous nos efforts, grande sera notre gratitude. This is $\zeta(2)$. In 1731 Euler found that

$$\sum \frac{1}{n^2} = (\log 2)^2 + \sum \frac{1}{2^{n-1}n^2}$$

and using this rapidly convergent series he calculated the sum to 6 decimals. The next year he got 20 decimals. In 1735 Euler proved that $\zeta(2)$ is exactly $\pi^2/6$. He also found all even Zeta values $\zeta(4)$, $\zeta(6)$, $\zeta(8)$, ... These are all irrational, yes, even transcendental numbers.

No formula revealing the true nature of the odd values $\zeta(2n+1)$ emerged. It was a sensation when R. Apéry in 1978 proved that $\zeta(3)$ is an irrational number, using the new formula

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{5}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3 \binom{2k}{k}}.$$

Here the progress stopped for a while. In 2000 T. Rivoal proved that among the values

$$\zeta(5),\ \zeta(7),\ \zeta(9), \ldots \zeta(2n+1), \ldots$$

there are infinitely many irrational numbers, without identifying even one of these. So far as I know, the best result by now is that at least one of the four values $\zeta(5)$, $\zeta(7)$, $\zeta(9)$, $\zeta(11)$ is irrational. The irrationality of $\zeta(5)$ is, as it were, still an open question! — This is what is new.