

Einstein and his theories of relativity

Øyvind G. Grøn

Forum for matematiske Perler
Trondheim, 16. september 2016

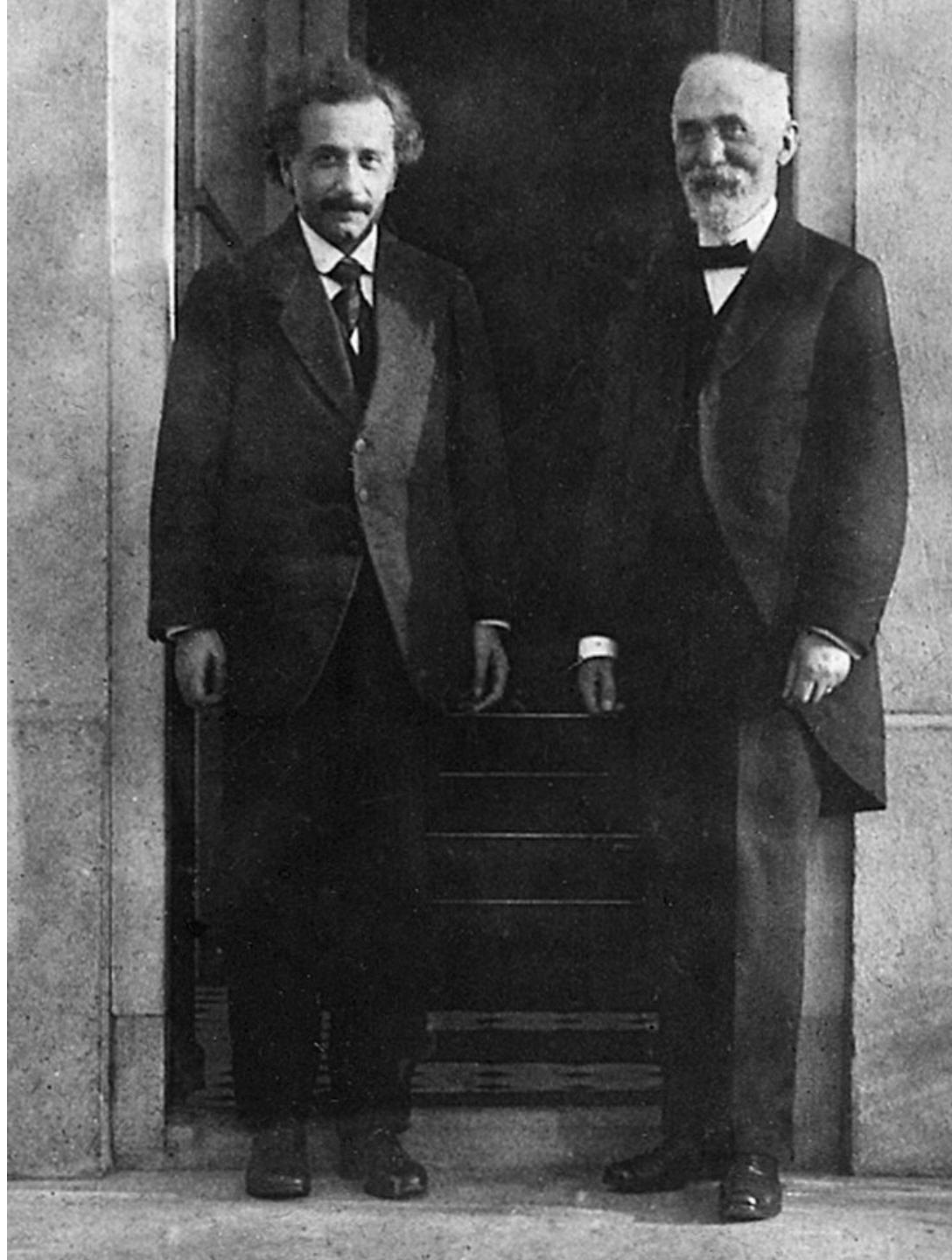
Albert Einstein 1879 – 1955

and

Hendrik Antoon Lorentz
1853-1928

Lorentz presented the
Lorentz transformations
between inertial frames.

But they were interpreted by him
in a non-relativistic manner,
in terms of an ether that was
absolutely at rest.



Henri Poincaré (1854 - 1912)

A polymath, he is known in mathematical circles as the Last Universalist due to the large number of significant contributions he made to various fields of mathematics and physics





Science is built up of facts, as a house is with stones. But a collection of facts is no more a science than a heap of stones is a house.

— *Henri Poincaré* —

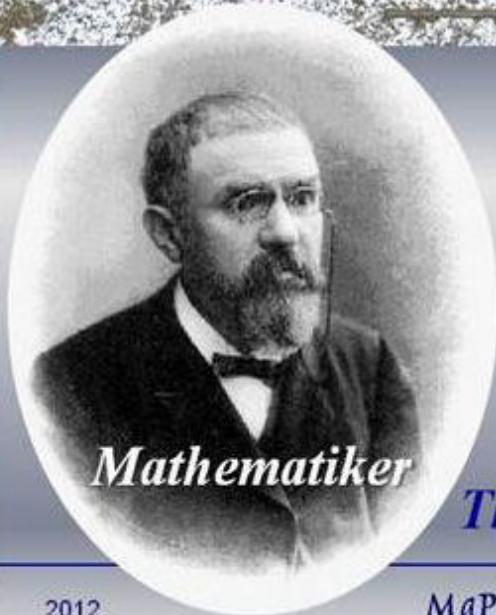
AZ QUOTES



"Mathematics is the art of giving the same name to different objects."

Henri Poincaré

HENRI POINCARÉ
1854 + 1912



Mathematiker

Theoretischer Physiker

Zum Gedenken



17. Juli 1912

2012

MaPhyPhil

Eg

MARKE INDIVIDUELL



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Both Lorentz and Poincaré showed that Maxwell's equations are invariant against a Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Poincaré and the Special Theory of Relativity

Supurna Sinha

In 1895 Poincaré wrote:

“Experiment has revealed a multitude of facts which can be summed up in the following statement: it is impossible to detect the absolute motion of matter, or rather the relative motion of ponderable matter with respect to the ether; all that one can exhibit is the motion of ponderable matter with respect to ponderable matter.”

But Poincaré was not ready to take the important step of eliminating the ether.

Poincaré and Einstein met only once, at the Solvay meeting of 1911.

On September 14th of 1904, Henri Poincaré (1854-1912) gives a talk at the congress of arts and science in Saint-Louis, Missouri¹, which title is *The Principles of Mathematical Physics* in which he defines :

« The principle of relativity, according to which the laws of physical phenomena must be the same for a stationary observer as for an observer carried along in a uniform motion of translation ; so that we have not and can not have any means of discerning whether or not we are carried along in such a motion. » (Poincaré, 1904, p306)

dynamics he tries to identify

In Poincaré's view, the principle of relativity is the consequence of a
under the hypotheses of an electromagnetic origin of inertia and of all forces.

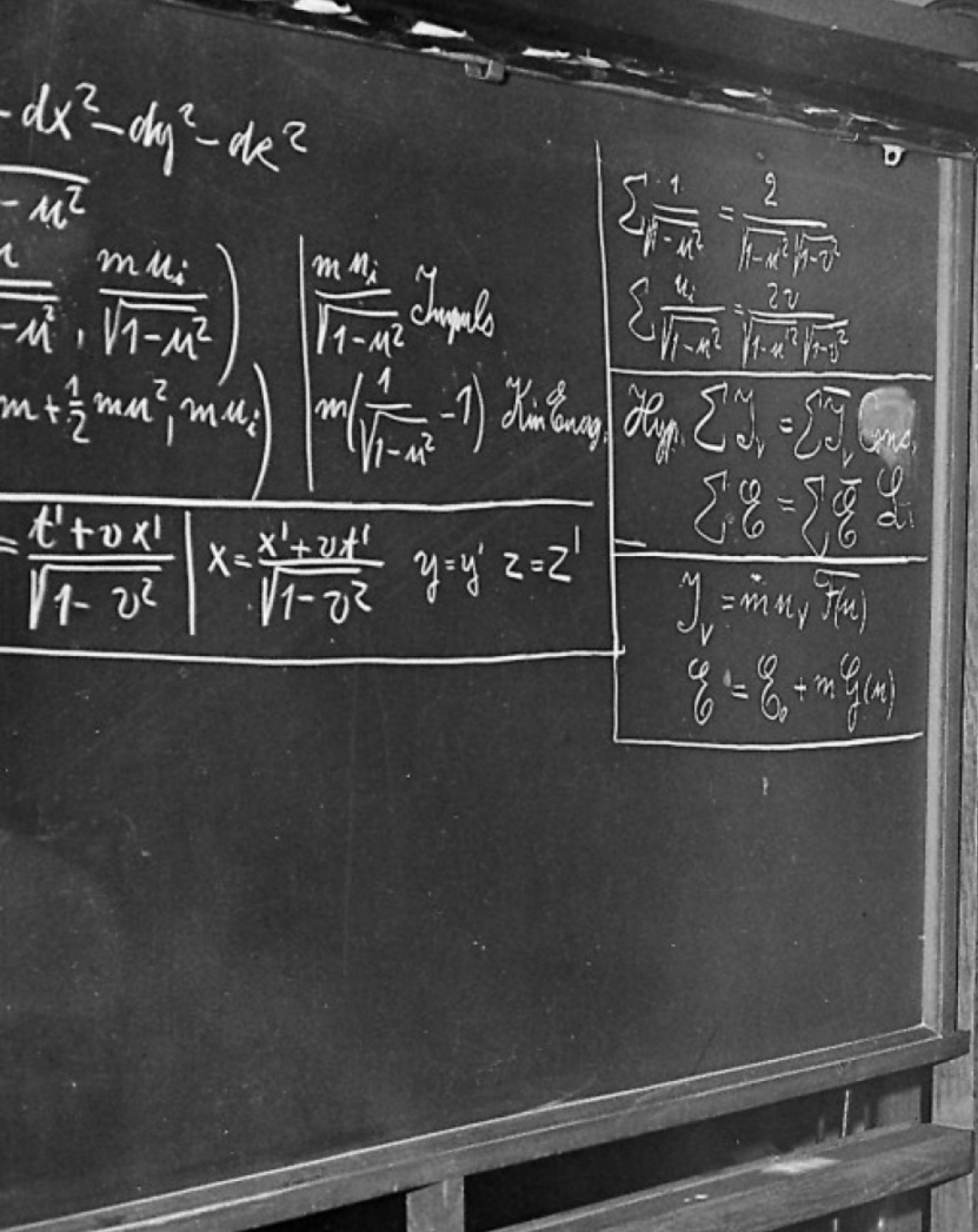
We saw that it is Poincaré who names and formulates the principle of relativity, names and corrects Lorentz transformations, reports and exploits its group structure. To these examples, we could add that he establishes the method for synchronizing clocks by light signals (*La mesure du temps*, Revue de métaphysique et de morale, T.6, janv 1898), the formula of additivity of velocities, the invariance of Maxwell's equations in a vacuum, and the hypothesis of the speed of light limit (Poincaré, 1905). Let's not forget that he also already uses a quadridimensional formalism that will inspire the future works of Minkowski, and then some. What is left ?

He clearly masters most of the concepts and technical tools of what we call now the special relativity theory, except (and it is fundamental !) that it is to him just **corrections** brought to Lorentz works, part of a **dynamics**, and what's more, **depending upon** Maxwell's electromagnetic theory.

That's what makes Einstein the real father of the theory, because he presents in his 1905 paper all of these points (except the importance of the group structure of Lorentz transformations) in a **coherent theory**, building a kinematics on which the laws of physics **will depend** (and not the other way around), including those of electromagnetism.



Einstein receives the relay baton from Poincaré in a relay where also Maxwell and Lorentz participated.



$$-dx^2 - dy^2 - dz^2$$

$$\left(\frac{m u_i}{\sqrt{1-u^2}}, m u_i \right) \quad \left| \begin{array}{l} \frac{m u_i}{\sqrt{1-u^2}} \text{ Impuls} \\ m \left(\frac{1}{\sqrt{1-u^2}} - 1 \right) \text{ Kin. Energy} \end{array} \right.$$

$$= \frac{t' + v x'}{\sqrt{1-v^2}} \quad \left| \quad x = \frac{x' + v t'}{\sqrt{1-v^2}} \quad y = y' \quad z = z' \right.$$

$$\sum \frac{1}{\sqrt{1-u^2}} = \frac{2}{\sqrt{1-u^2} \sqrt{1-v^2}}$$

$$\sum \frac{u_i}{\sqrt{1-u^2}} = \frac{2v}{\sqrt{1-u^2} \sqrt{1-v^2}}$$

$$\text{Hyp. } \sum \mathcal{L}_v = \sum \mathcal{L}_v \text{ (cons.)}$$

$$\sum \mathcal{E} = \sum \mathcal{E} \text{ (L. d.)}$$

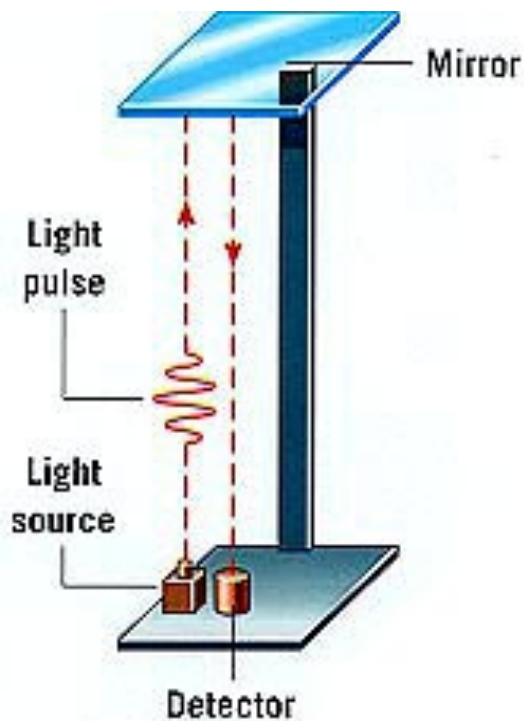
$$y_v = m u_v \overline{F(u)}$$

$$\mathcal{E} = \mathcal{E}_0 + m \mathcal{E}(u)$$



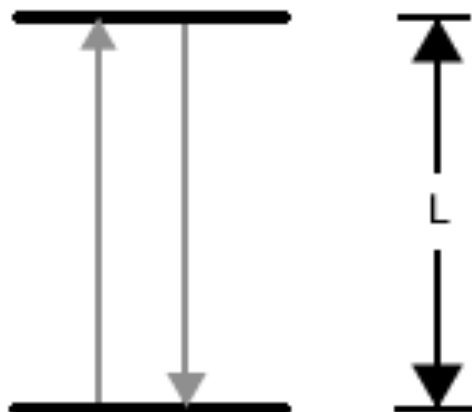
Albert Einstein 1879 - 1955

En av relativitetsteoriens konsekvenser er at legemer (klokker) som beveger seg, eldes langsommere enn legemer som er i ro.

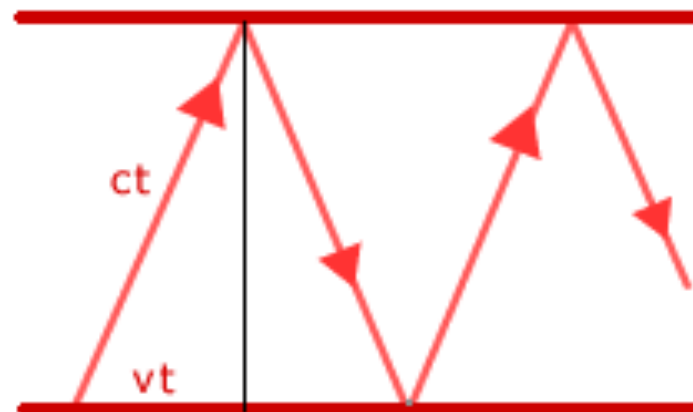


Lysklokke i tog

PATHWAY OF LIGHT ON TRANSVERSE CLOCK



At absolute rest

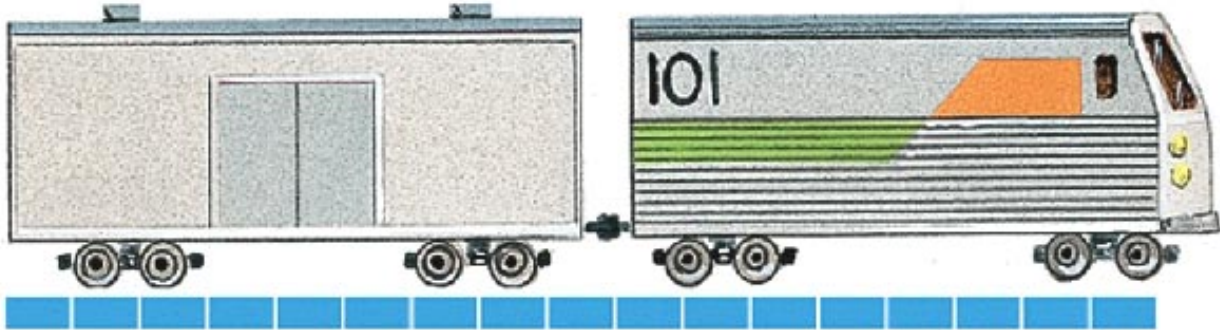


On a moving inertial frame

observert fra toget

observert fra perrongen

Lengdekontraksjon



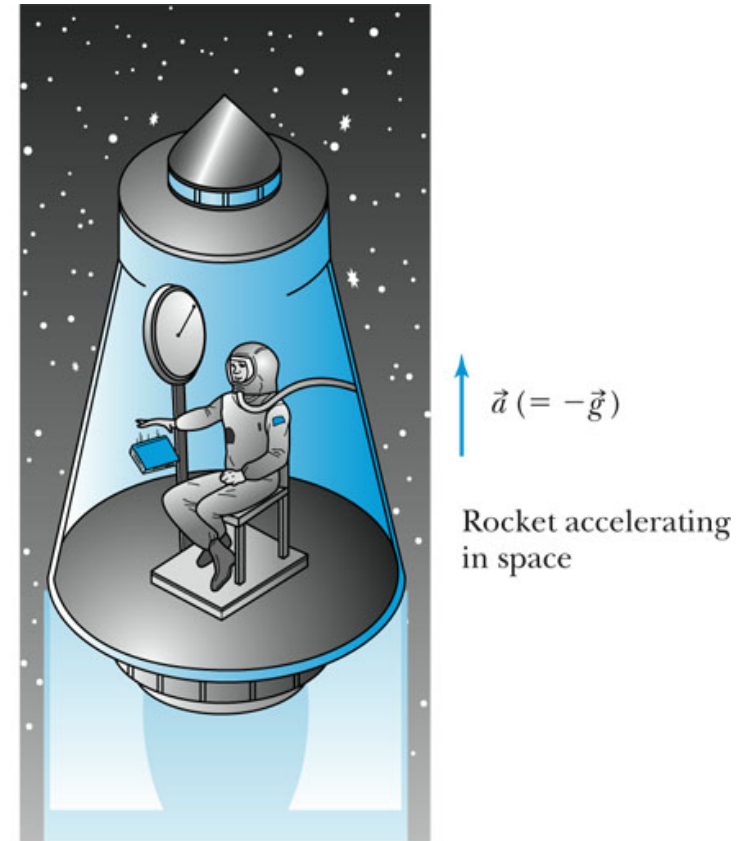
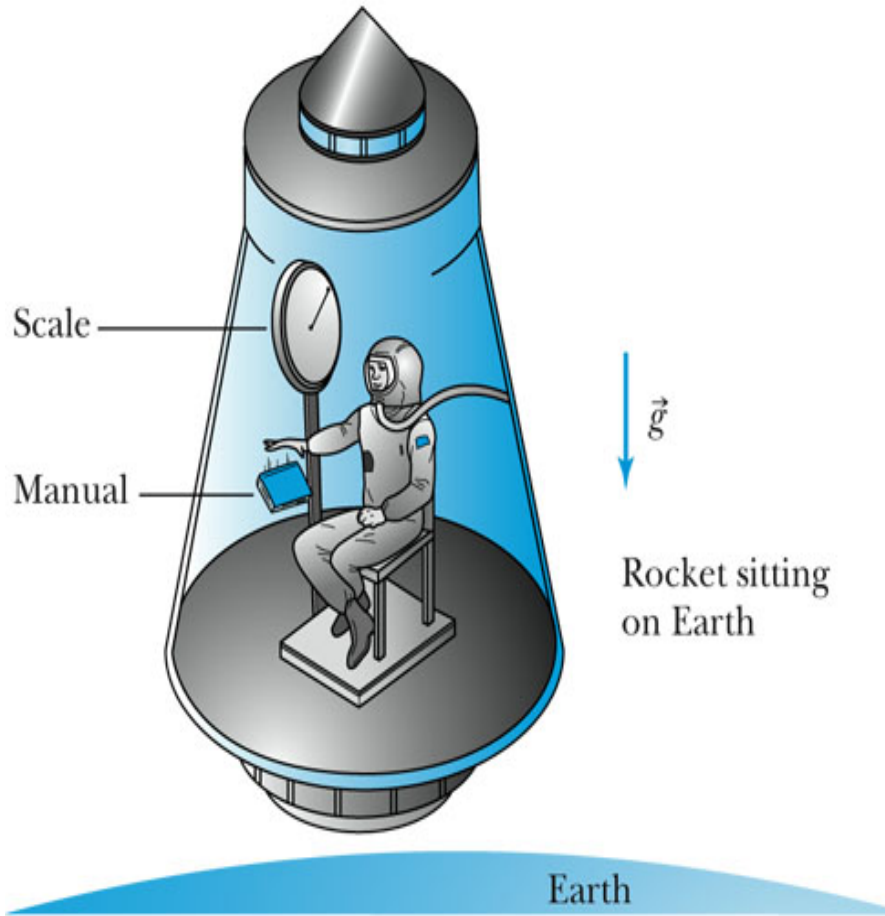
At rest



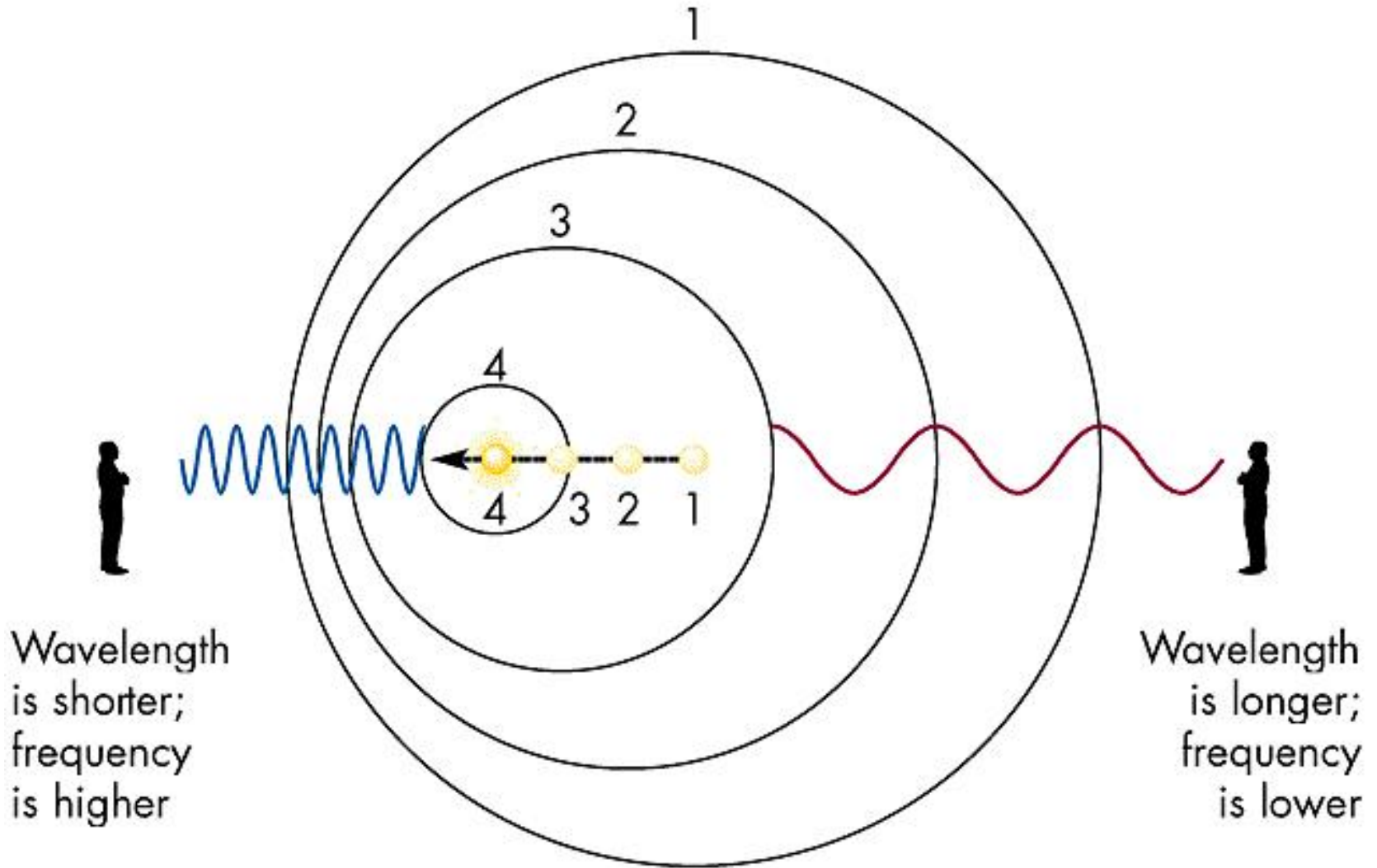
In motion

Målt ved samtidighet er avstanden mellom forreste og bakerste ende av en gjenstand kortere jo raskere gjenstanden beveger seg.

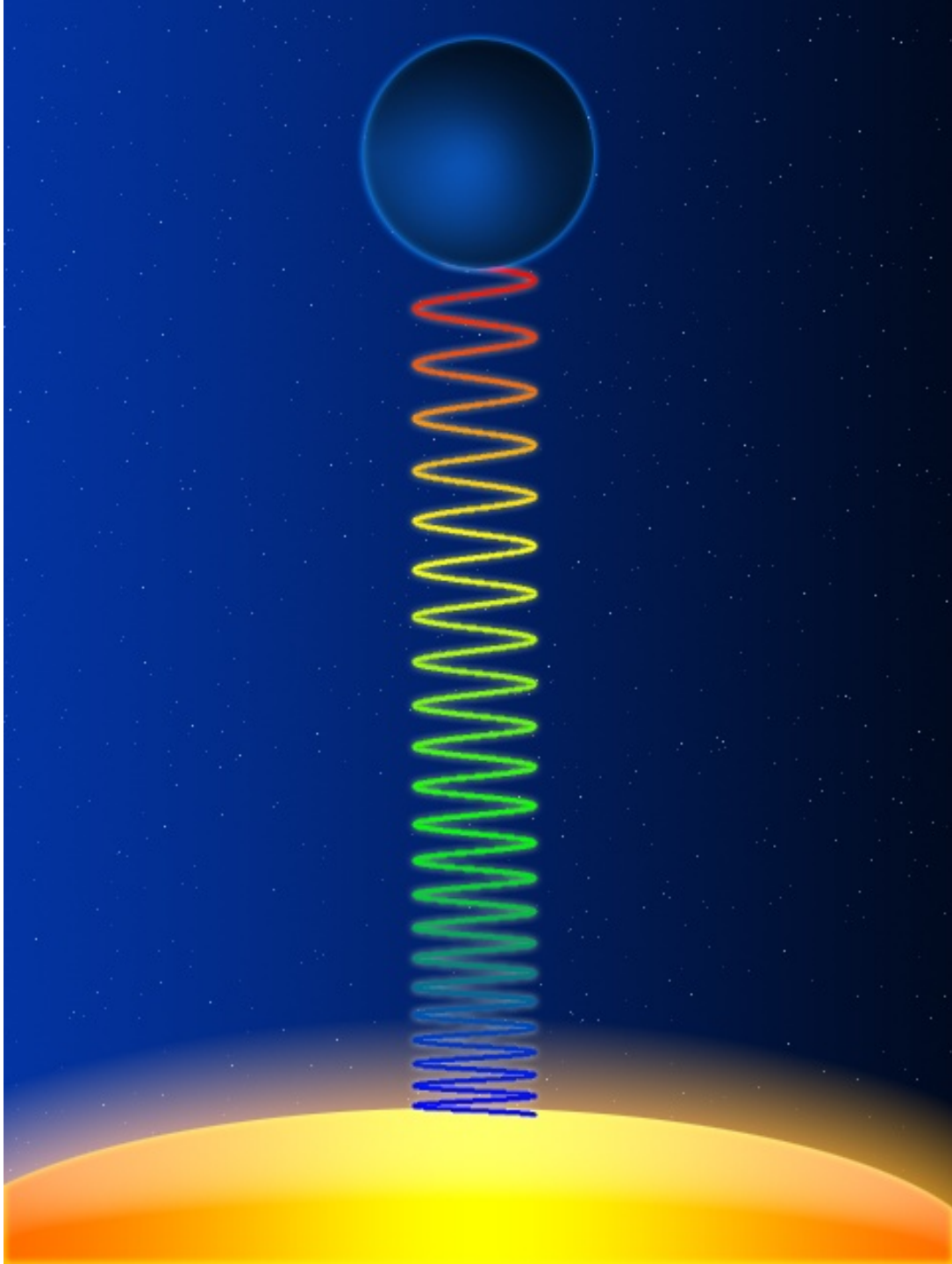
The principle of equivalence



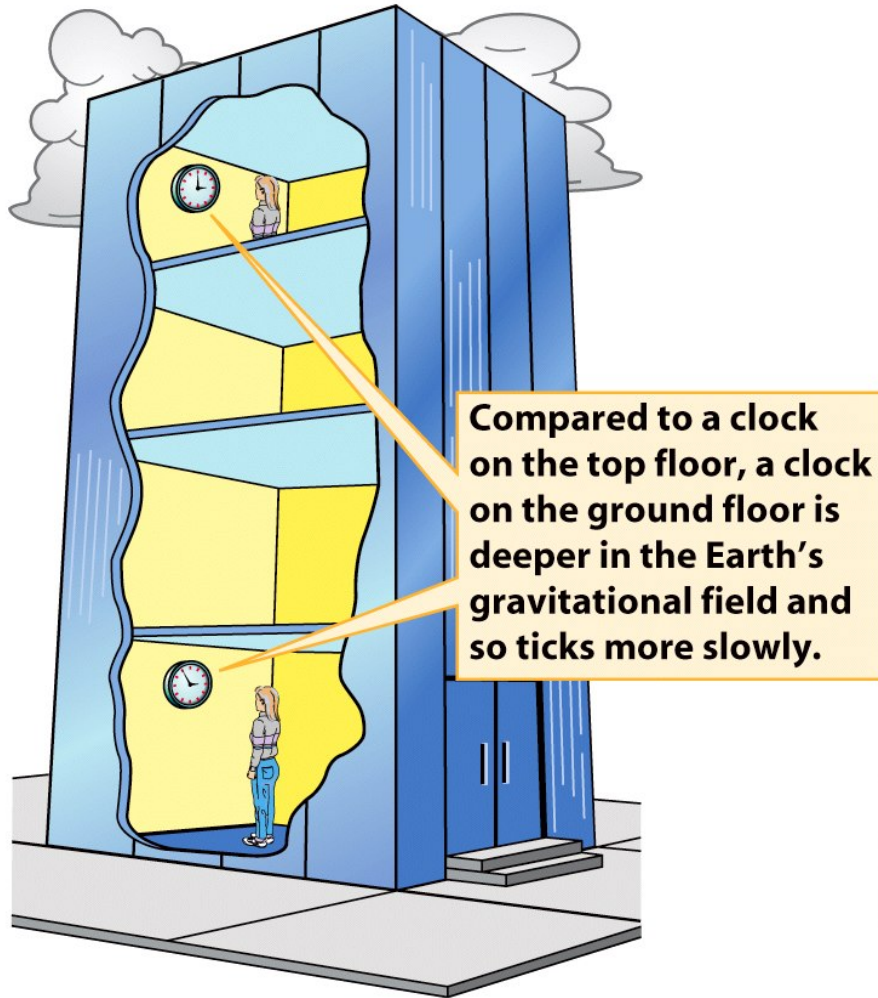
The Doppler effect



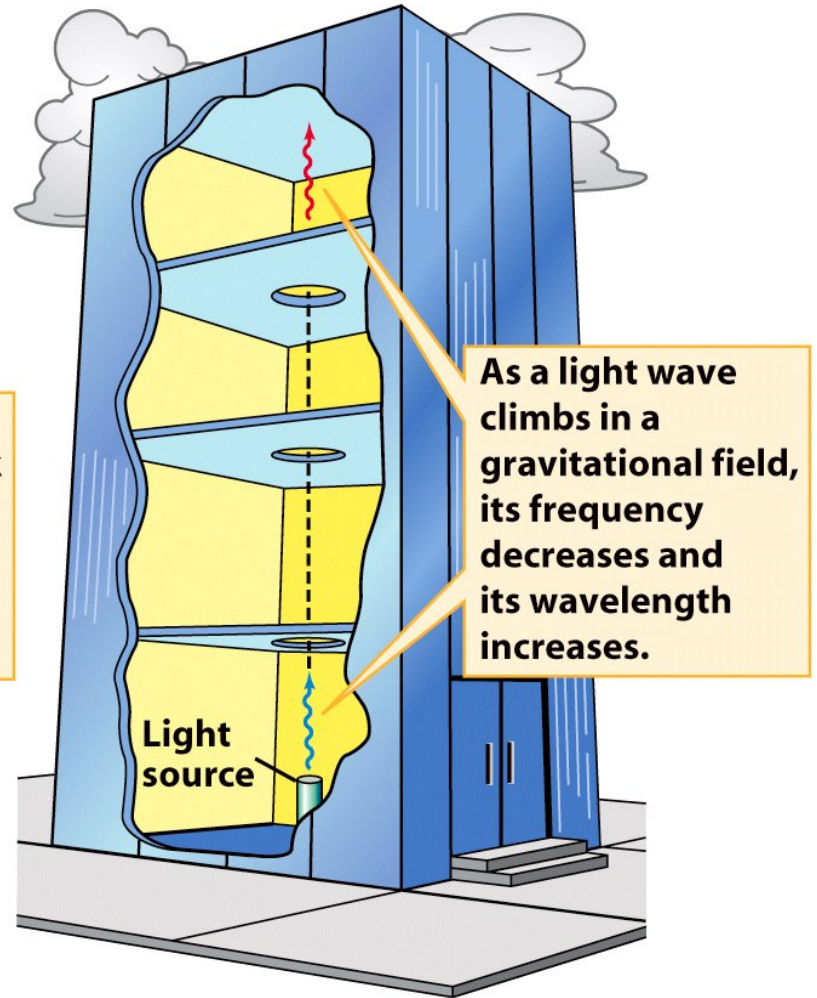
The numbers on the circles show wave fronts emitted at the events 1, 2, 3, 4



Gravitational time dilation and red shift

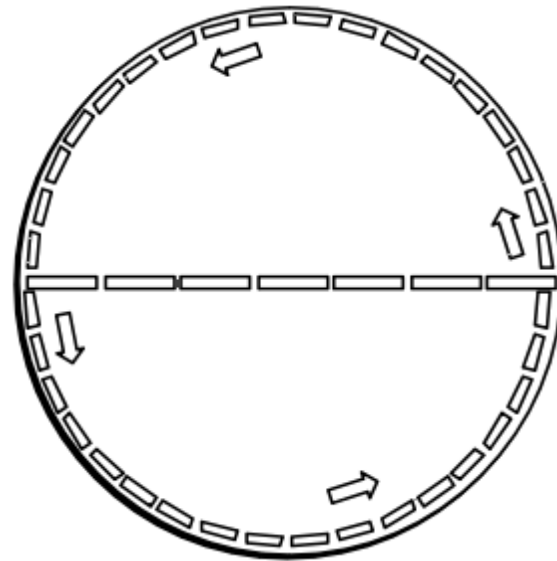
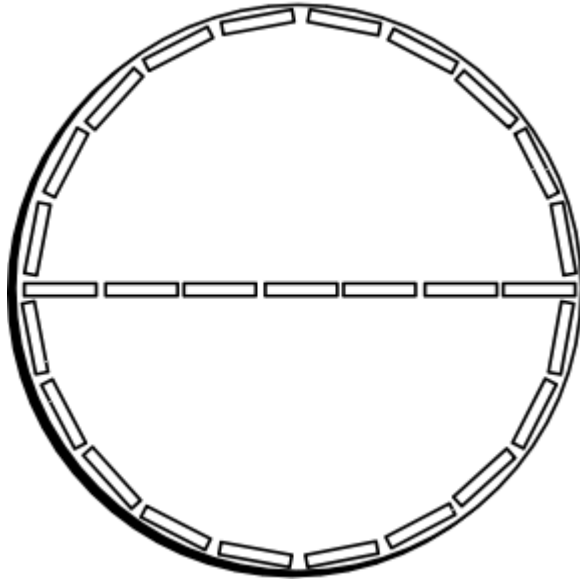


(a) The gravitational slowing of time

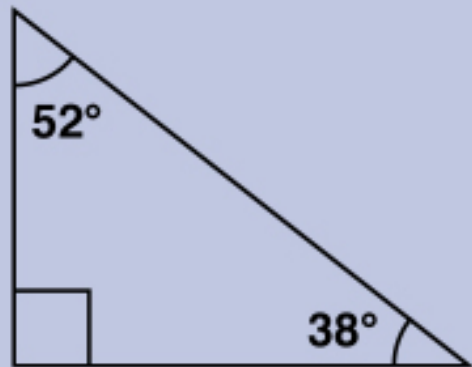


(b) The gravitational redshift

Geometri on a rotating disk

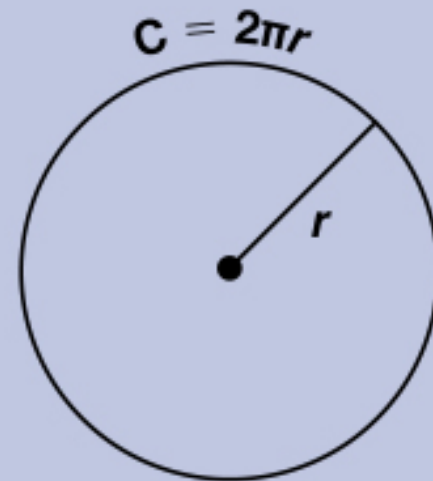
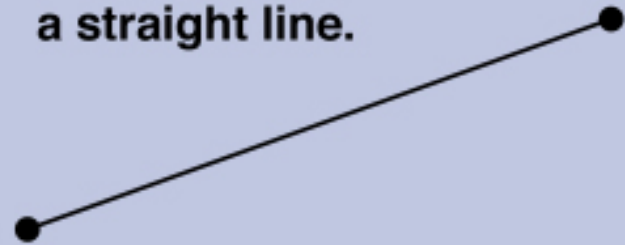


The sum of the angles in a triangle is equal to 180° .



Lines that are parallel somewhere are parallel everywhere.

The shortest distance between two points is a straight line.



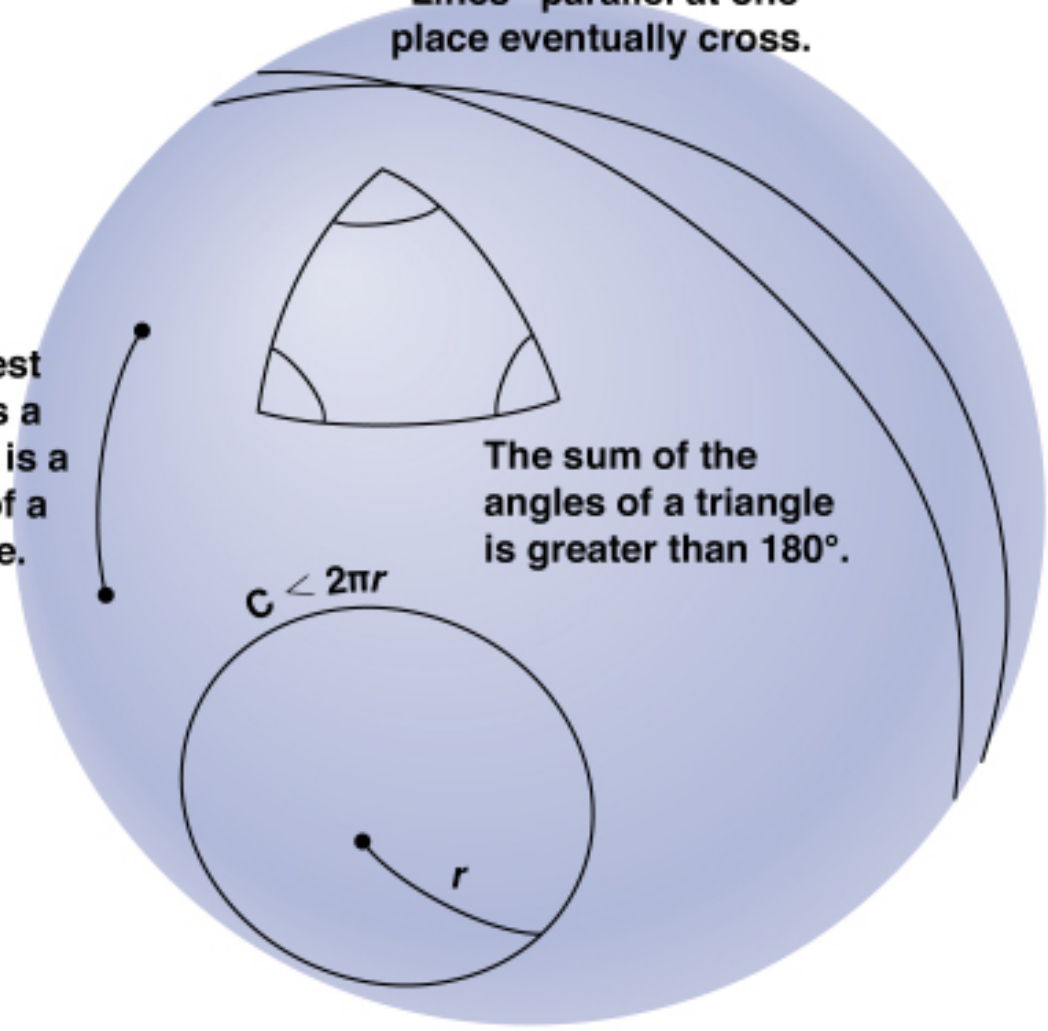
"Lines" parallel at one place eventually cross.

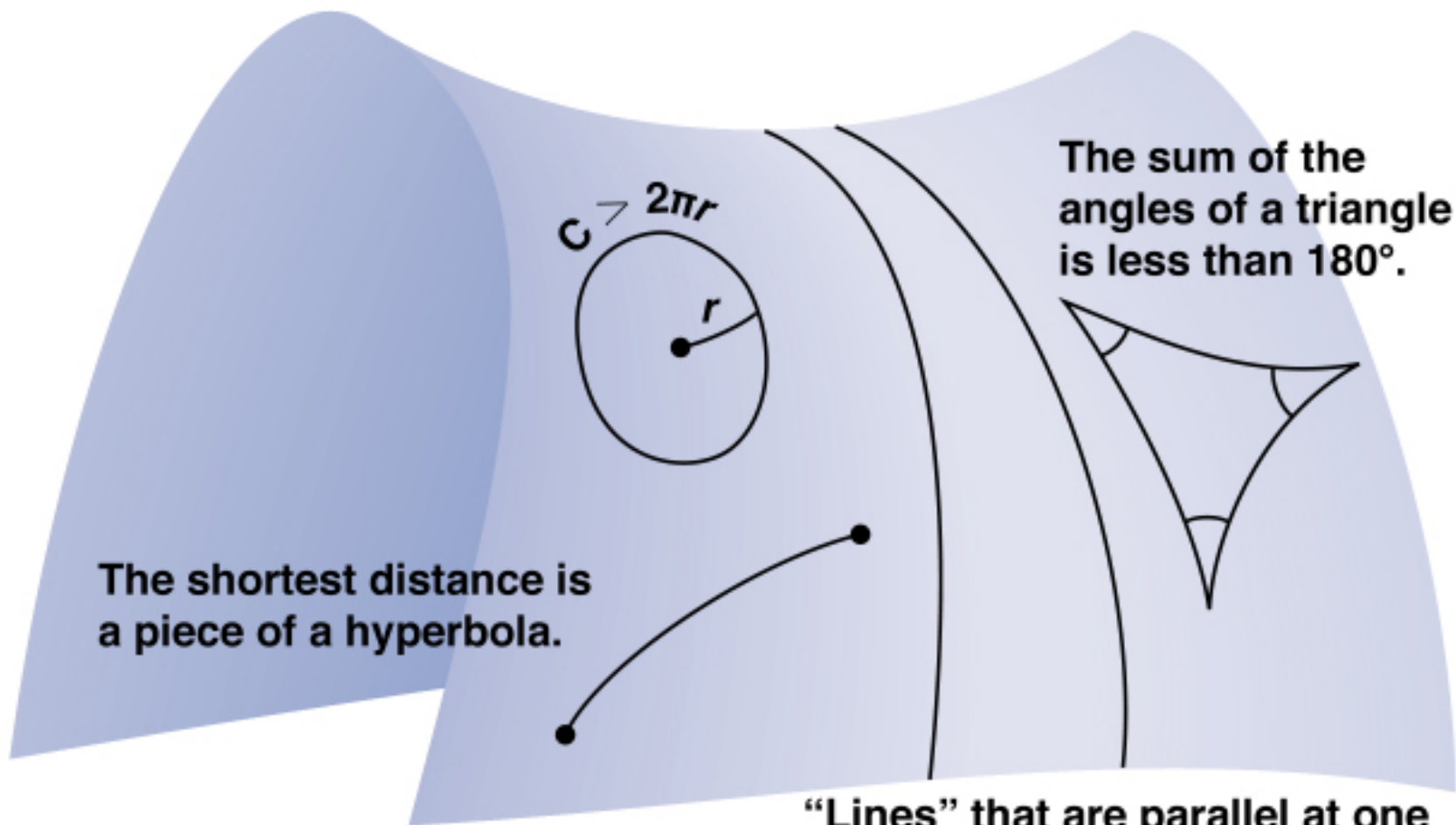
The shortest distance is a curve that is a segment of a great circle.

The sum of the angles of a triangle is greater than 180°.

$C < 2\pi r$

r





The shortest distance is a piece of a hyperbola.

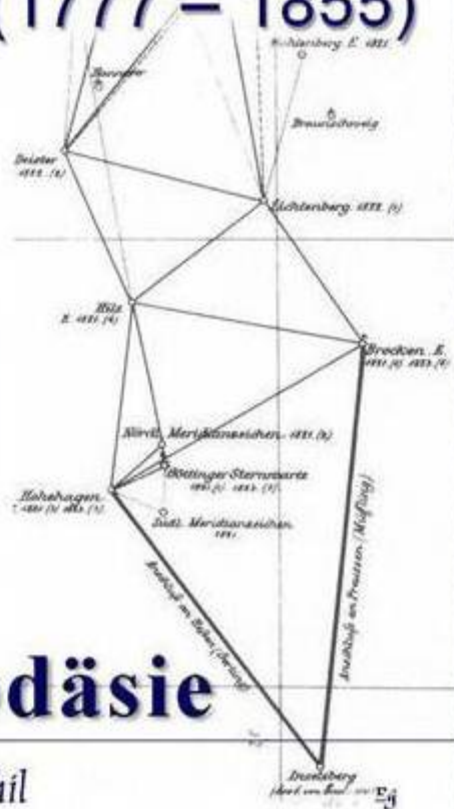
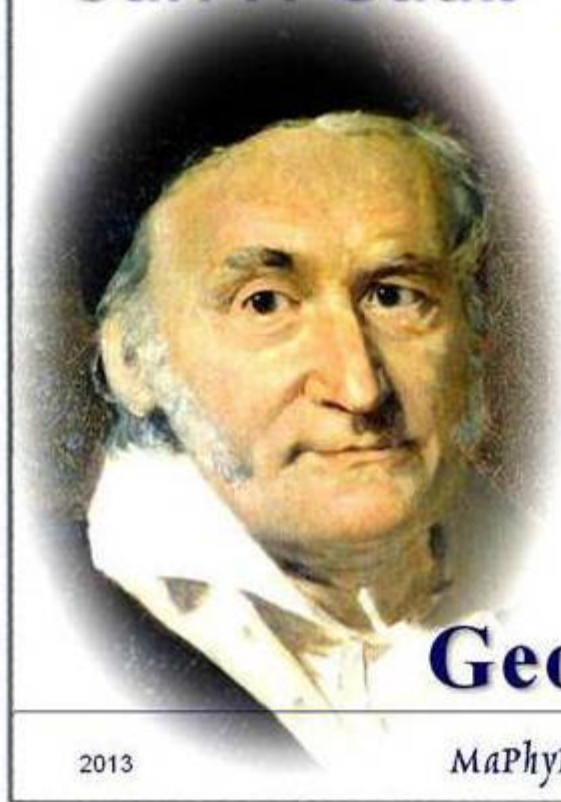
The sum of the angles of a triangle is less than 180°.

“Lines” that are parallel at one place eventually diverge.



18 years old Gauss proved that a seventeen sided regular polygon can be constructed by ruler-and-compass

Carl F. Gauß (1777 – 1855)



Geodäsie

MaPhyPhil

2013

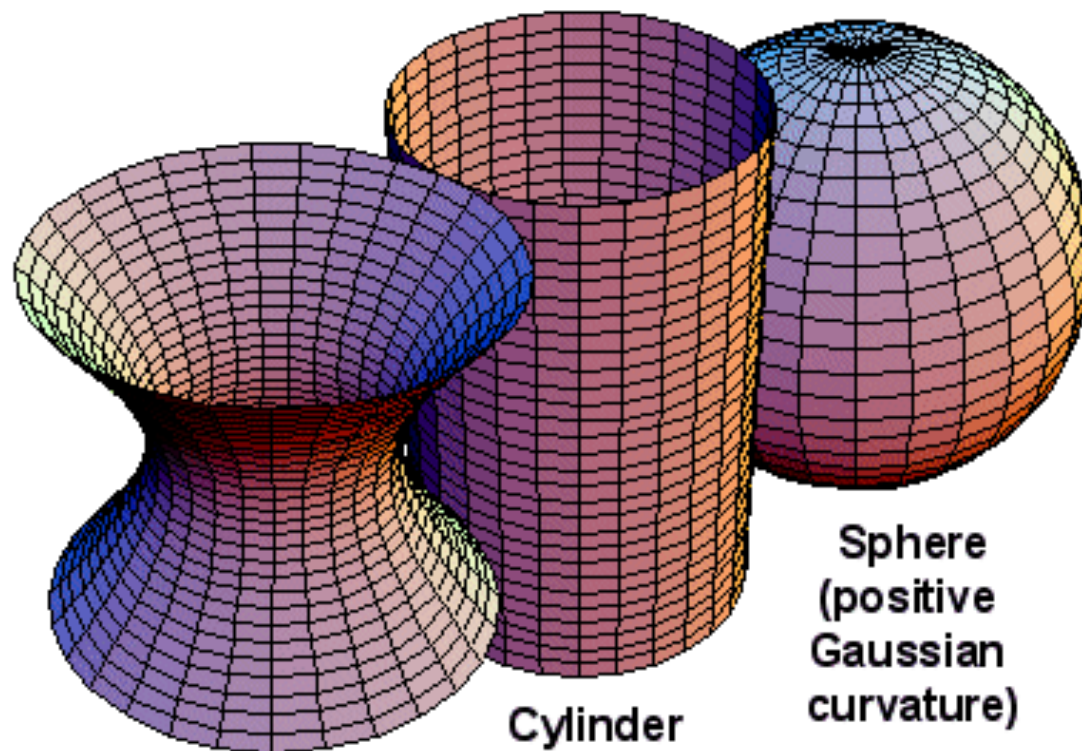
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**Hyperboloid
(negative
Gaussian
curvature)**

**Cylinder
(zero
Gaussian
curvature)**

**Sphere
(positive
Gaussian
curvature)**

Bernhard Riemann (1826 – 1866)

$$ds^2 = \sum_{i,k}^{1\dots n} g_{ik} dx_i dx_k$$

\mathbb{R}^n



*n-dimensionale
Mannigfaltigkeiten*

2010

MaPhyPhil

5g



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BRIEFMARKE INDIVIDUELL



Marcel Grossman 1912.



Einstein and Hilbert



David Hilbert 1862 - 1943

Einstein visited David Hilbert at the University of Göttingen in June and July 1915. Hilbert was very impressed by Einstein's theory but Hilbert being very energetic and brilliant he raced Einstein to find the covariant forms of the equations. This created tremendous pressure on Einstein. Between June 1915 and November 25, 1915 Einstein was a driven man. He worked with superhuman intensity until he finally found the covariant forms of his equations. Hilbert came up with the covariant equations at the same time as Einstein did.

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Allgemeine Relativitätstheorie

20. Nov. 1915

Königliche Akademie zu Göttingen :

$$\sqrt{g} (R_{ik} - \frac{1}{2} R g_{ik}) = - \frac{\partial \sqrt{g} L}{\partial g_{ik}}$$

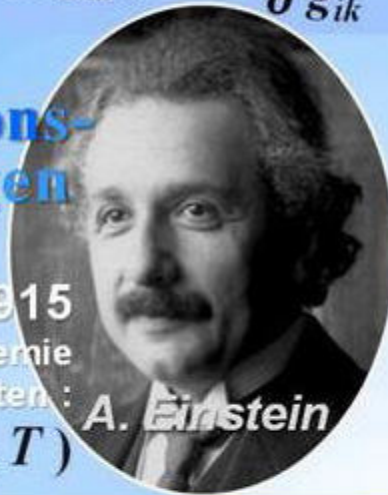


D. Hilbert

Gravitationsgleichungen

25. Nov. 1915

Preußische Akademie
der Wissenschaften :



A. Einstein

$$R_{ik} = - \gamma (T_{ik} - \frac{1}{2} g_{ik} T)$$

2012

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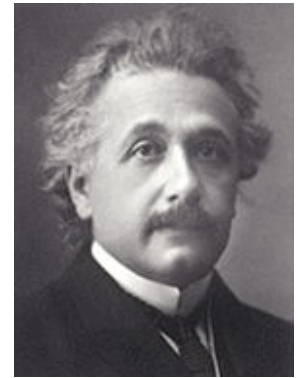
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BRIEFMARKE INDIVIDUELL

Albert Einstein, 1915



The general theory of relativity:

- Spacetime is generally curved. Non Euclidean geometry.

The line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- Matter and energy curve spacetime.

Einstein's field equations:

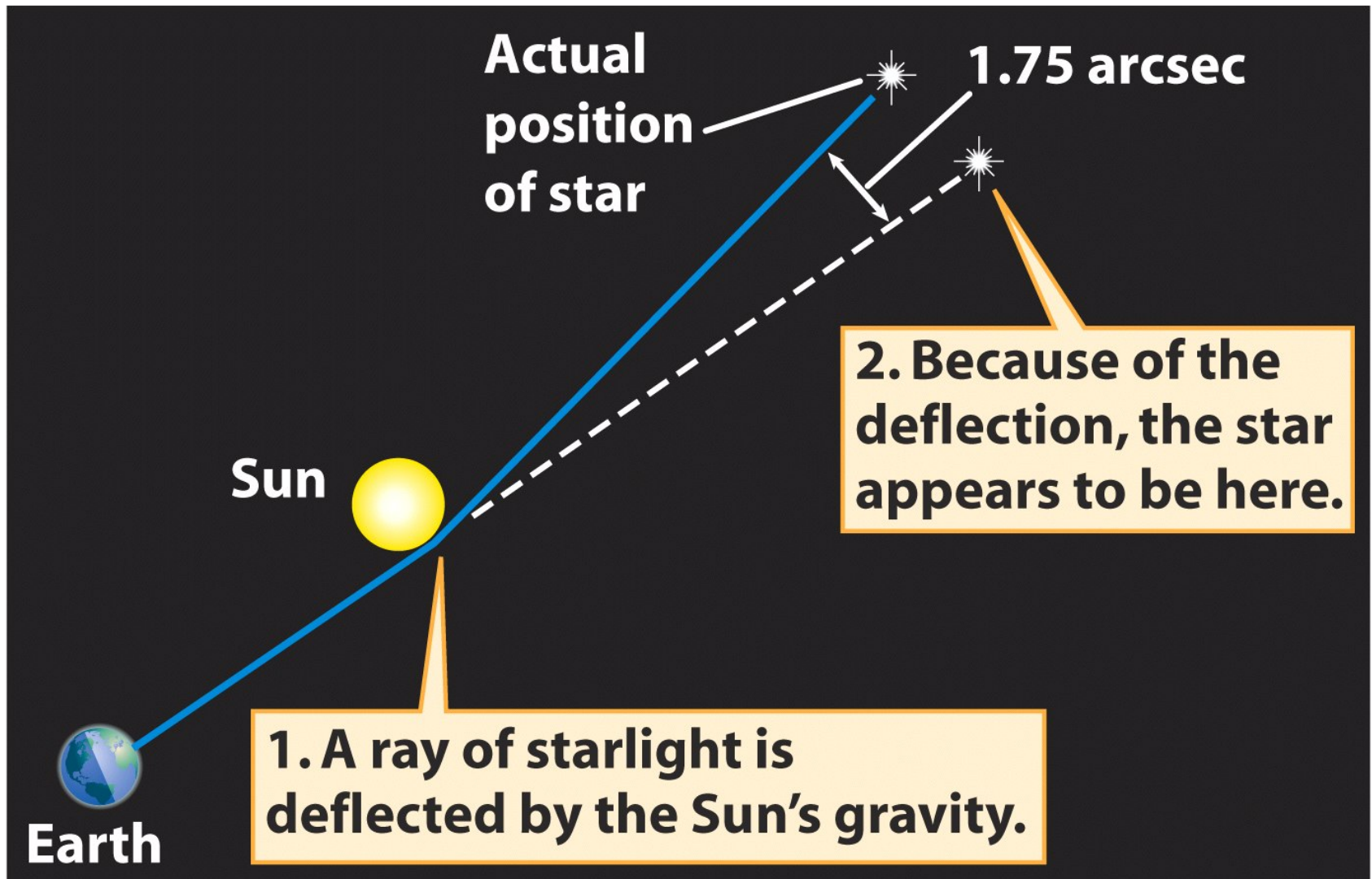
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Curved spacetime tells matter how to move according to the equation of geodesic curves.

The geodesic equation:

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\lambda\nu}^\mu \frac{dx^\lambda}{ds} \frac{dx^\nu}{ds} = 0$$

Gravitational light deflection



LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less
Agog Over Results of Eclipse
Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed
or Were Calculated to be,
but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could
Comprehend It, Said Einstein When
His Daring Publishers Accepted It.

Actual Position of the Star
Distance from the Earth to the Stella Background is more than 93,000,000,000,000 miles.

Apparent Position of the Star

THE SUN
Distance from the Earth 93,000,000 miles

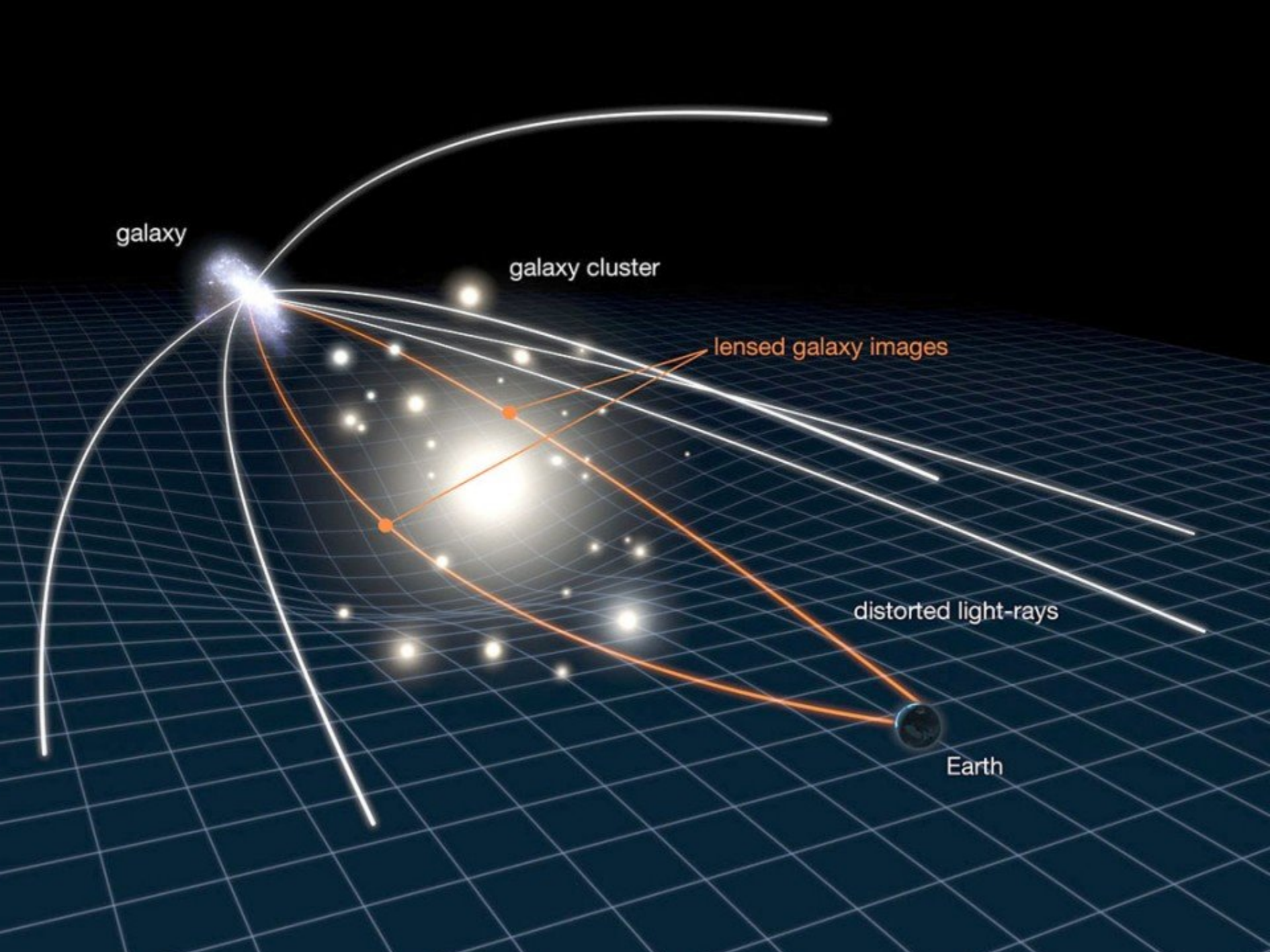
This Diagram shows the proportional Displacement of the Stars in relation to the distance from the Sun. The amount of Displacement is exaggerated about 500 times.

Apparent Position: ↑
Actual Position: ★

Showing Path of Total Eclipse of May 28-29, 1919, and positions of the two Observation Stations.

THE OBSERVATION STATION AT SOBRAL, IN BRAZIL

The Corona



galaxy

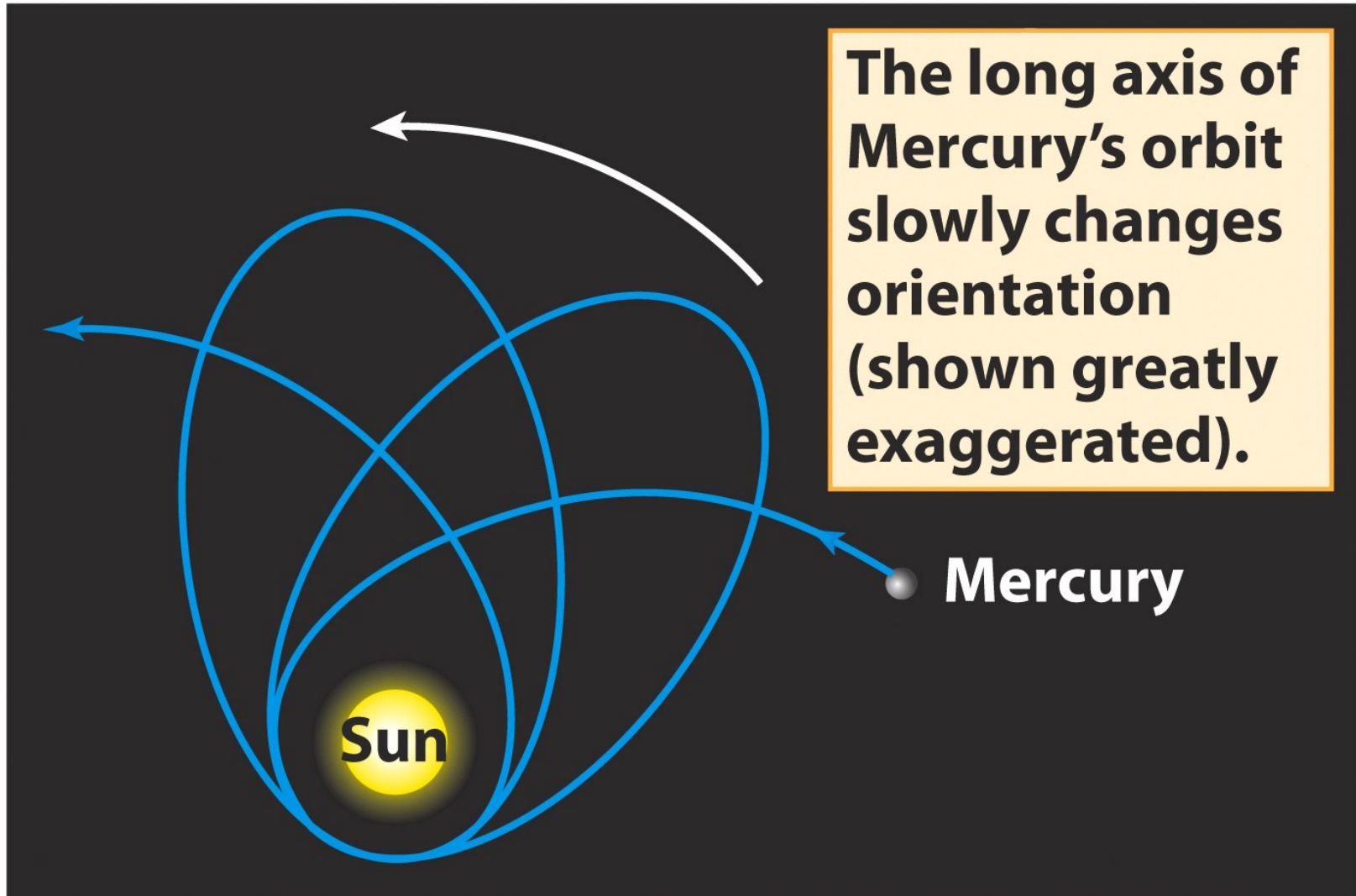
galaxy cluster

lensed galaxy images

distorted light-rays

Earth

The perihelion precession of Mercury



When forced to summarize the general theory of relativity in one sentence; *time* and *space* and *gravity* have no separate existence from *matter*

- Albert Einstein



UNIVERSITY OF TEXAS

Wheeler

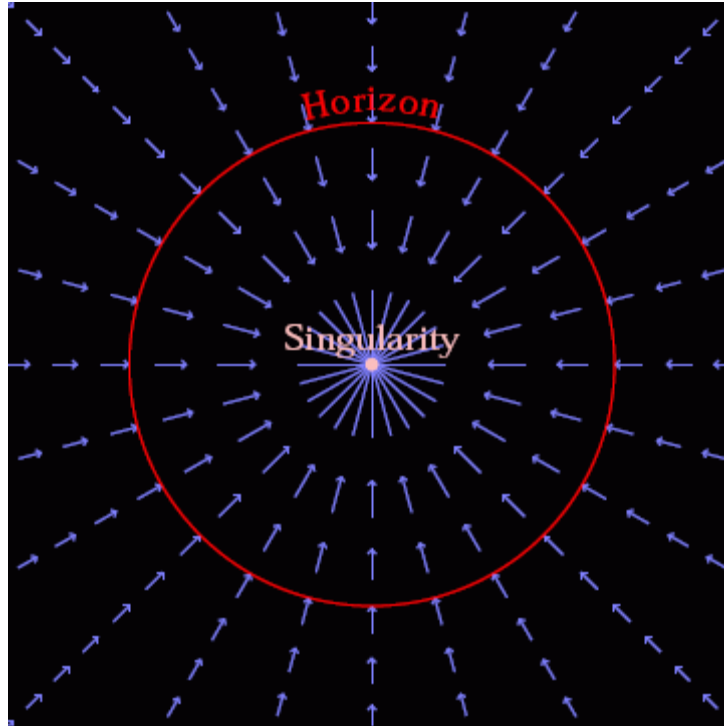
Matter tells *space*
how to curve
Space tells *matter*
how to move

John Archibald Wheeler (1911 – 2008)

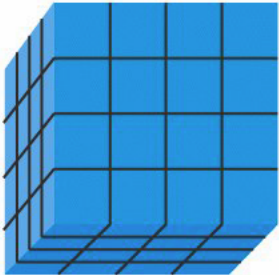
A river model of space

Simen Braeck and Øyvind Grøn

inspired by an article by Andrew J. S. Hamilton and Jason P. Lisle

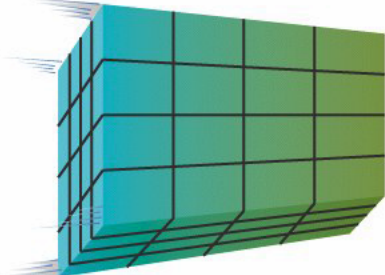


Probe far from black hole

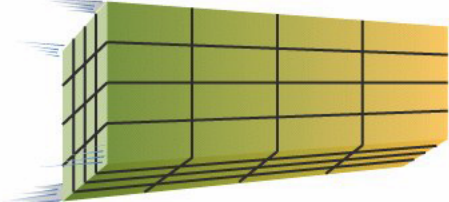


(a)

Probe approaching black hole



(b)



(c)

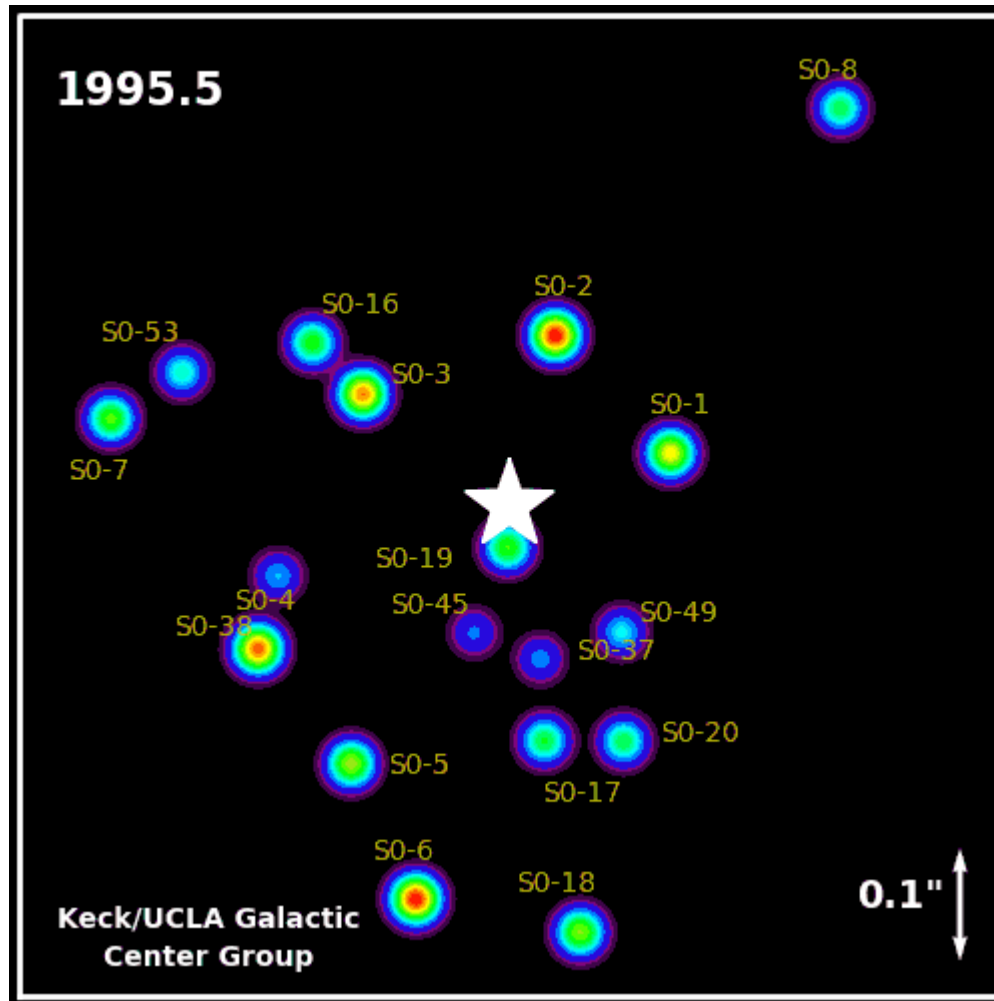


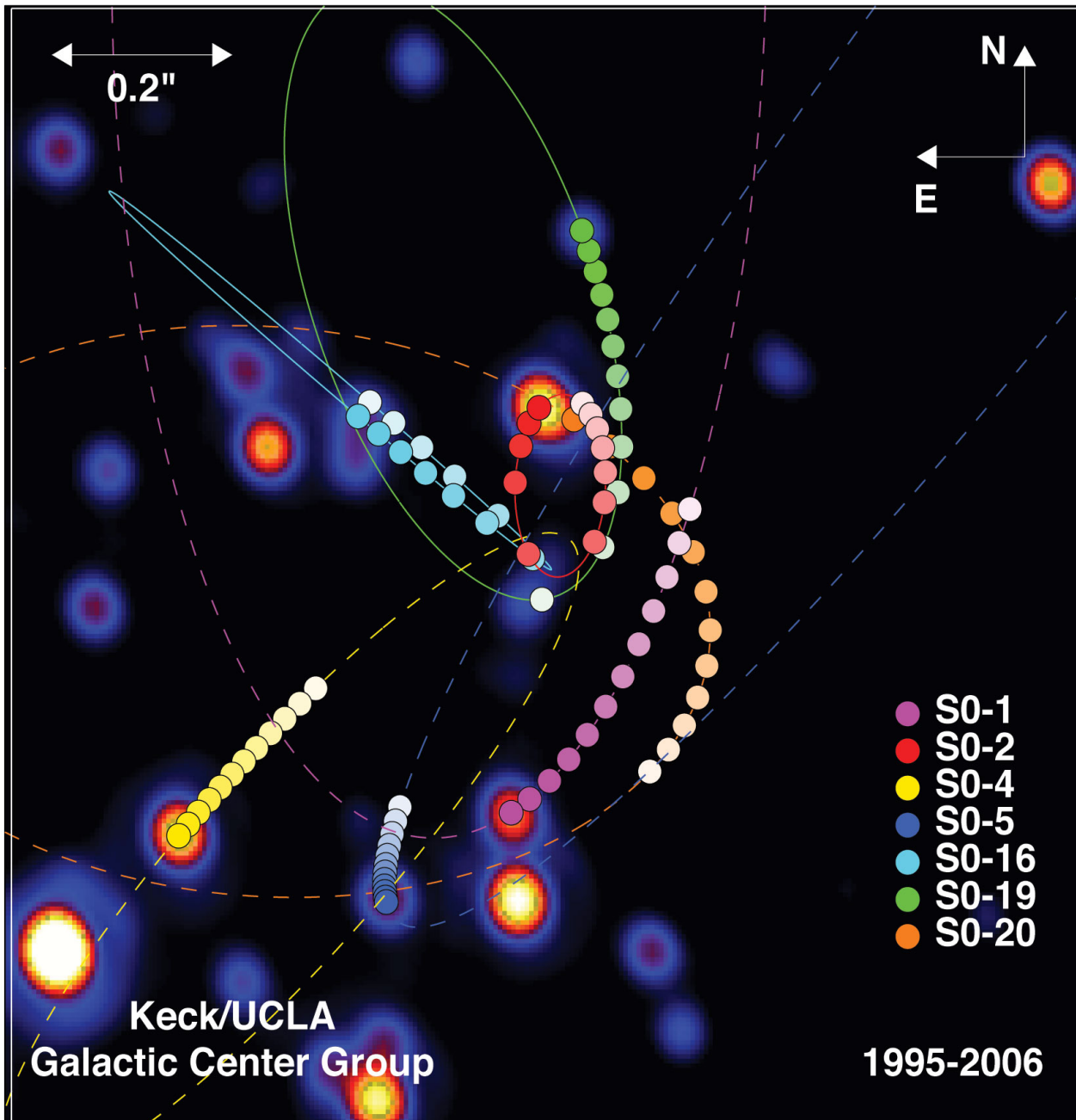
(d)

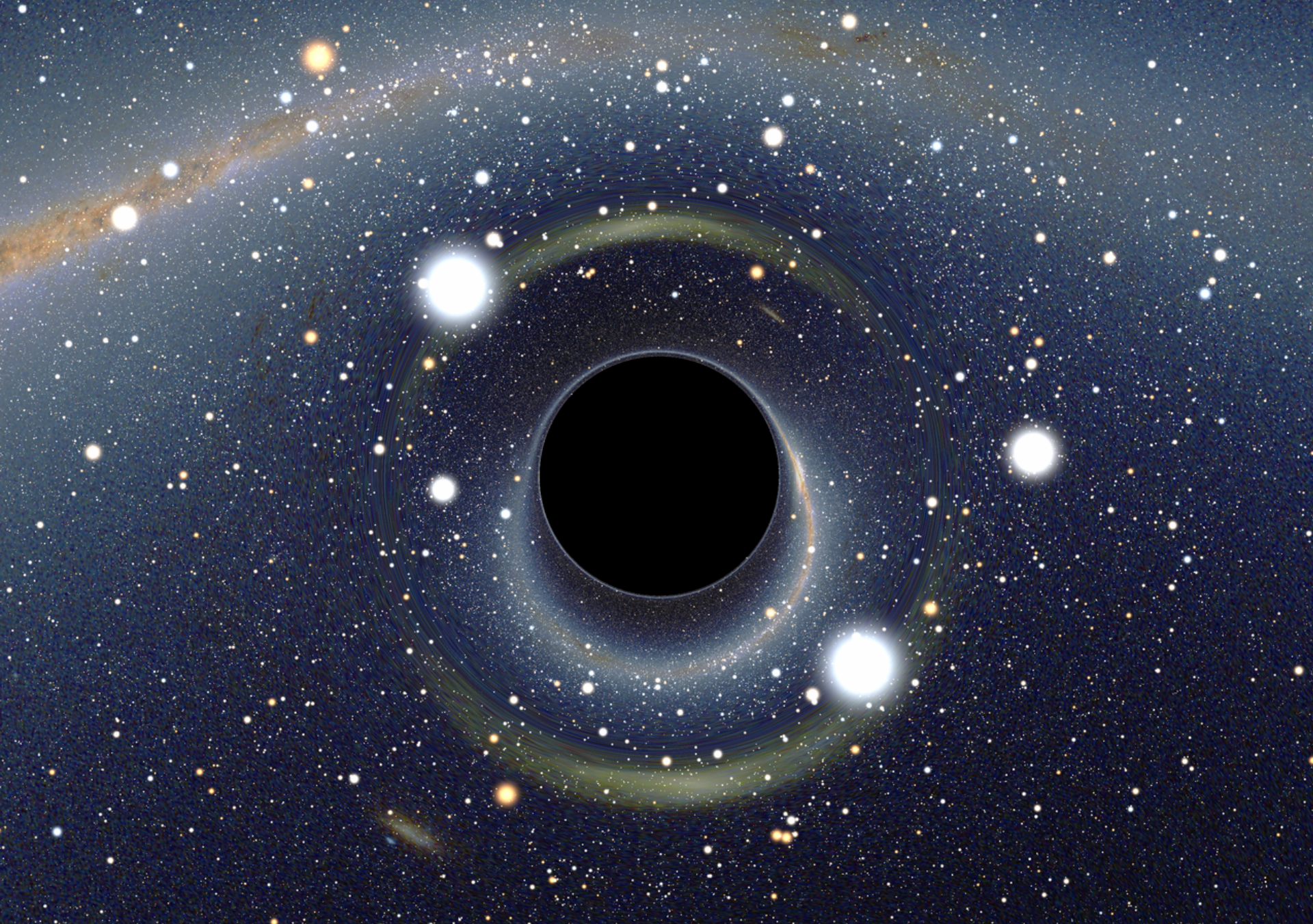
Black hole
Event horizon

A black circle representing a black hole. A line points from the text 'Event horizon' to the circle.

The motion of the stars close to the center of the Milky way

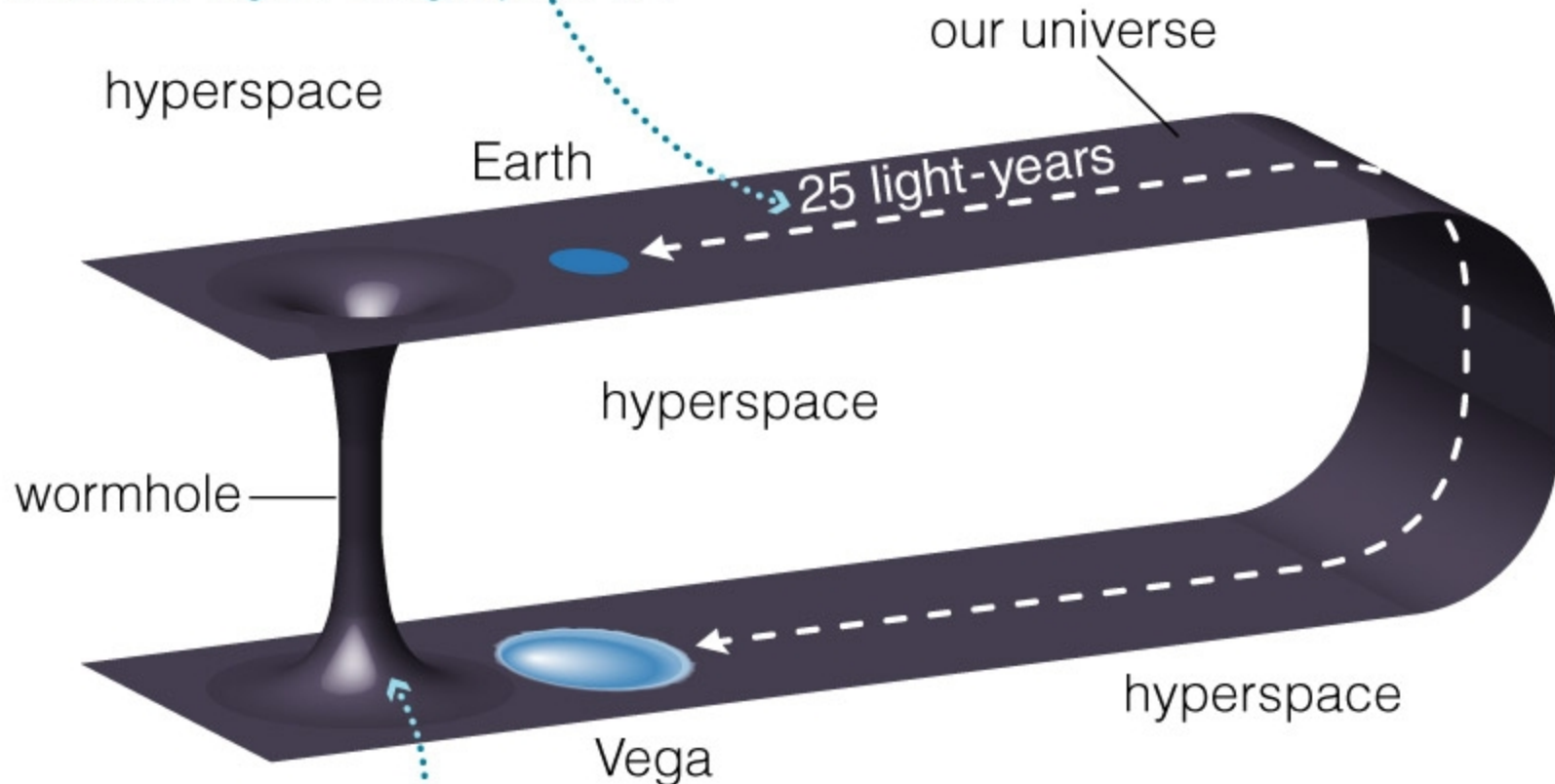




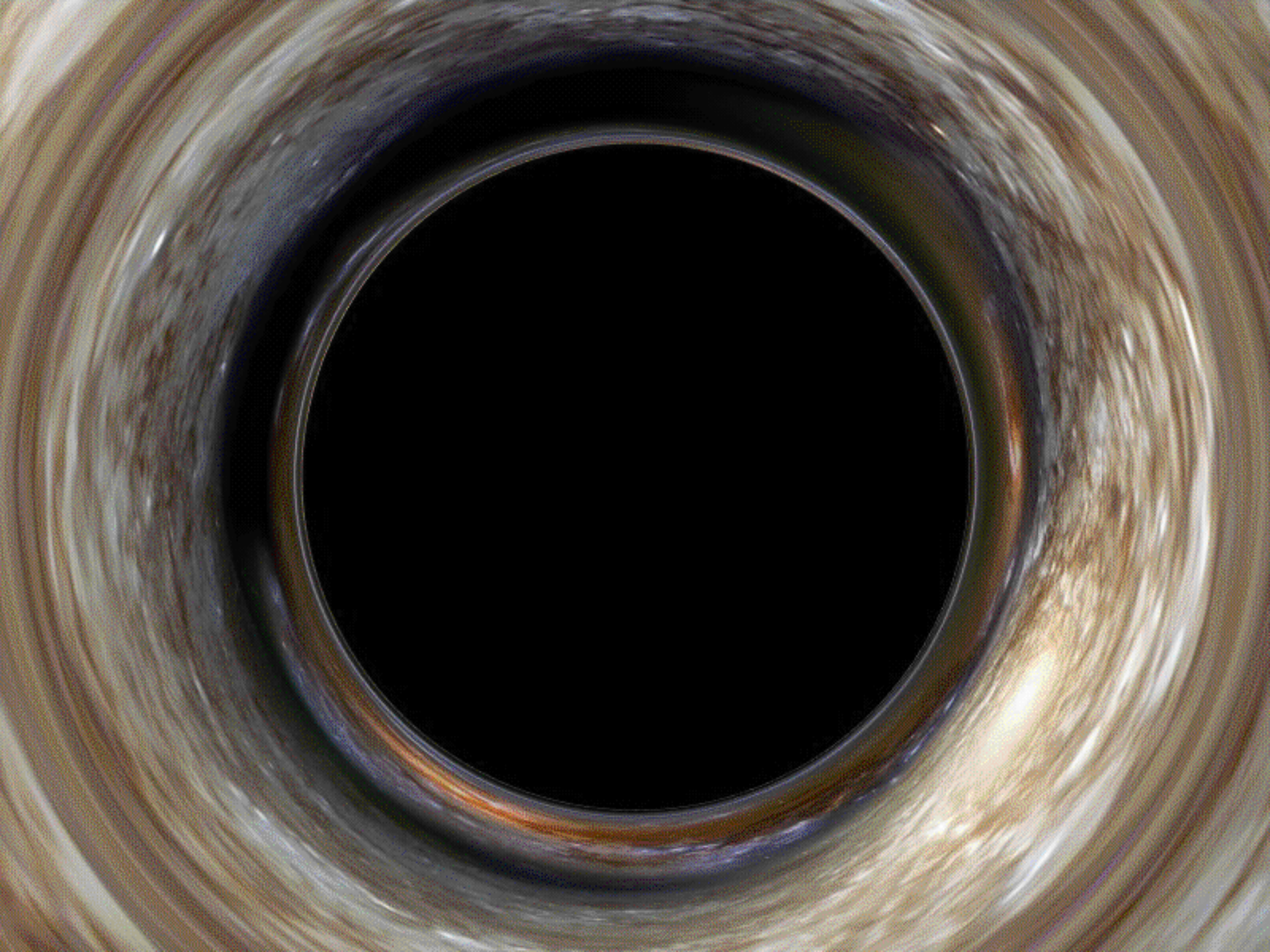


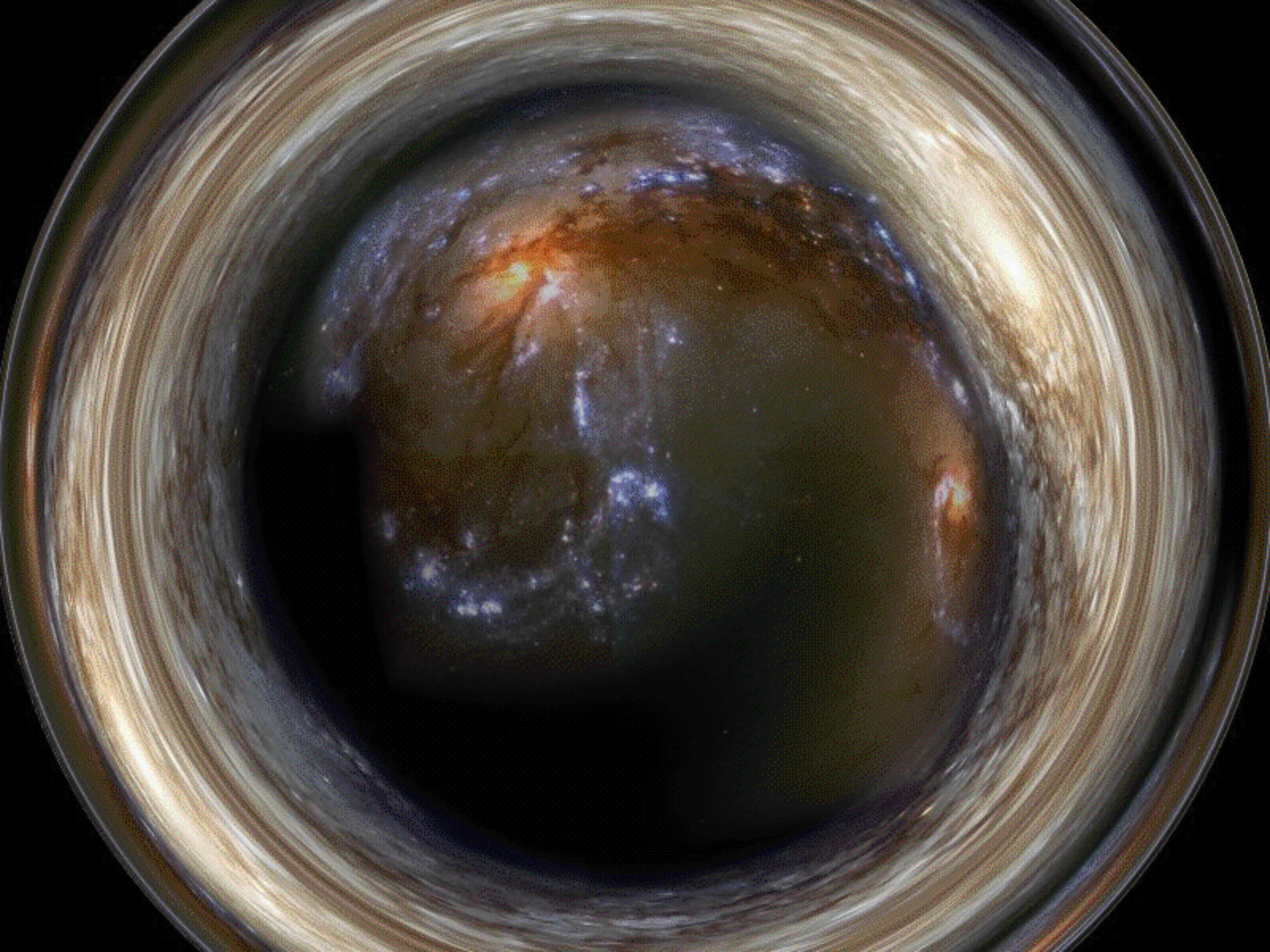
Computer generated illustration of a black hole

The distance through our universe between Earth and Vega is 25 light-years . . .



. . . but the distance would be much shorter if we could travel through a wormhole.







Panasonic
FS EDGED

AS SERIOUS
AS YOU CAN
GET

Sidewalks
of New York



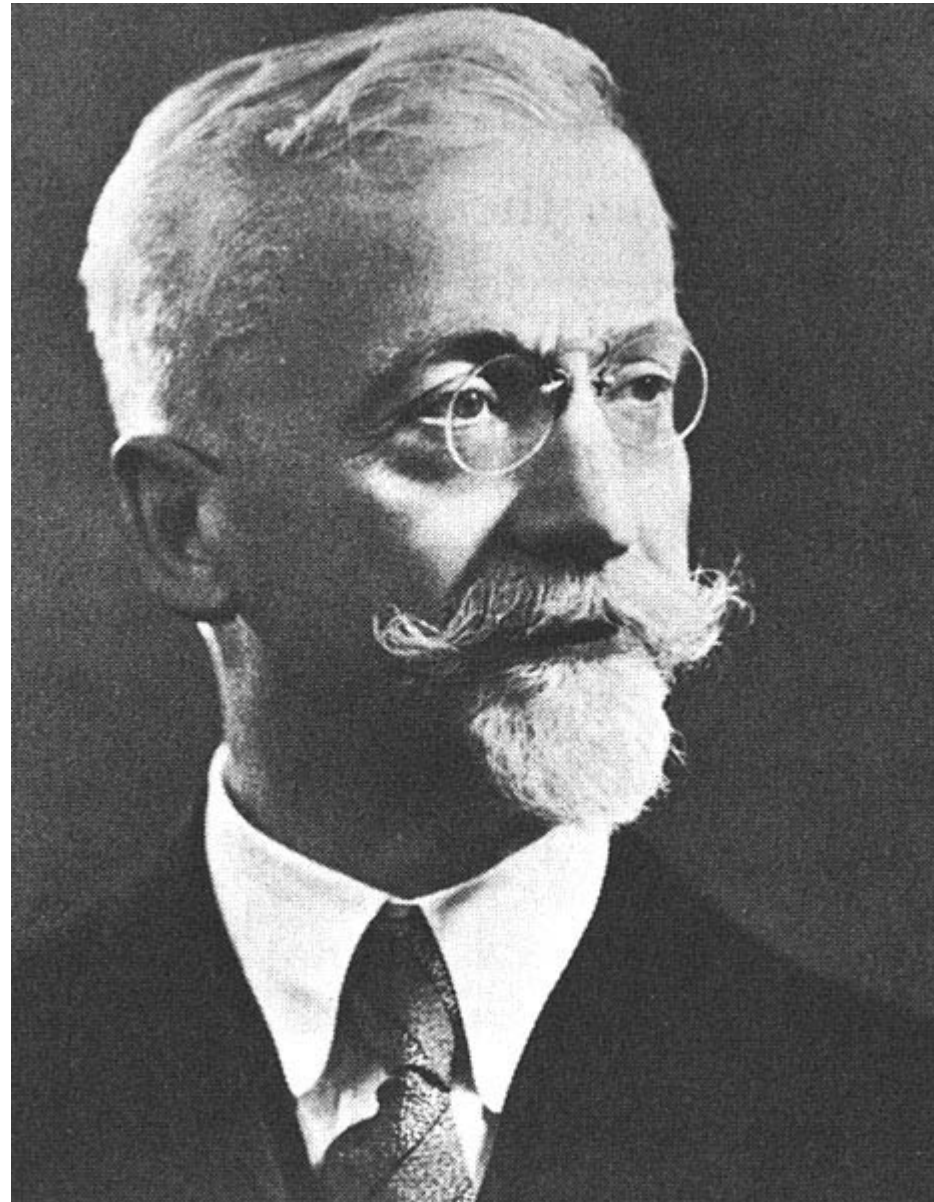
Worth 1000.com
Worth Status: STABLE

Elie Cartan 1869 – 1951

Cartan constructed
an antisymmetric tensor formalism:
The theory of differential forms.

He formulated Einstein's equations
in terms of forms.

Also he generalized Einstein's
General theory of relativity
by constructing a new theory
where spacetime not only
had curvature, but also
a new property called *torsion*,
a sort of twist.



Emmy Noether 1882 - 1935

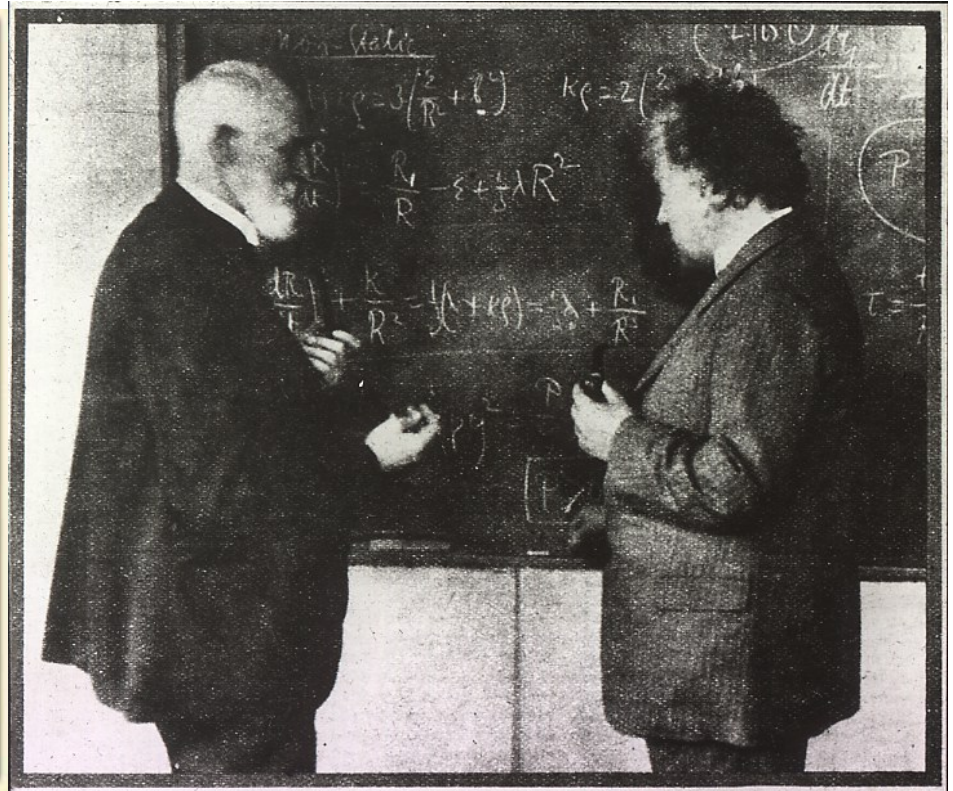
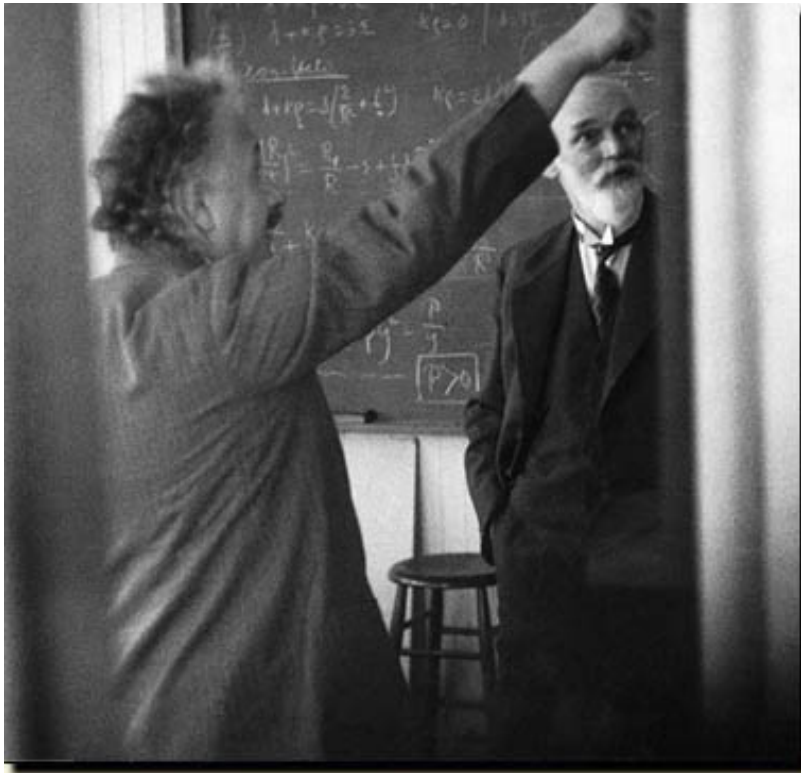
Emmy Noether proved that
to every symmetry
there belongs
a conserved quantity.





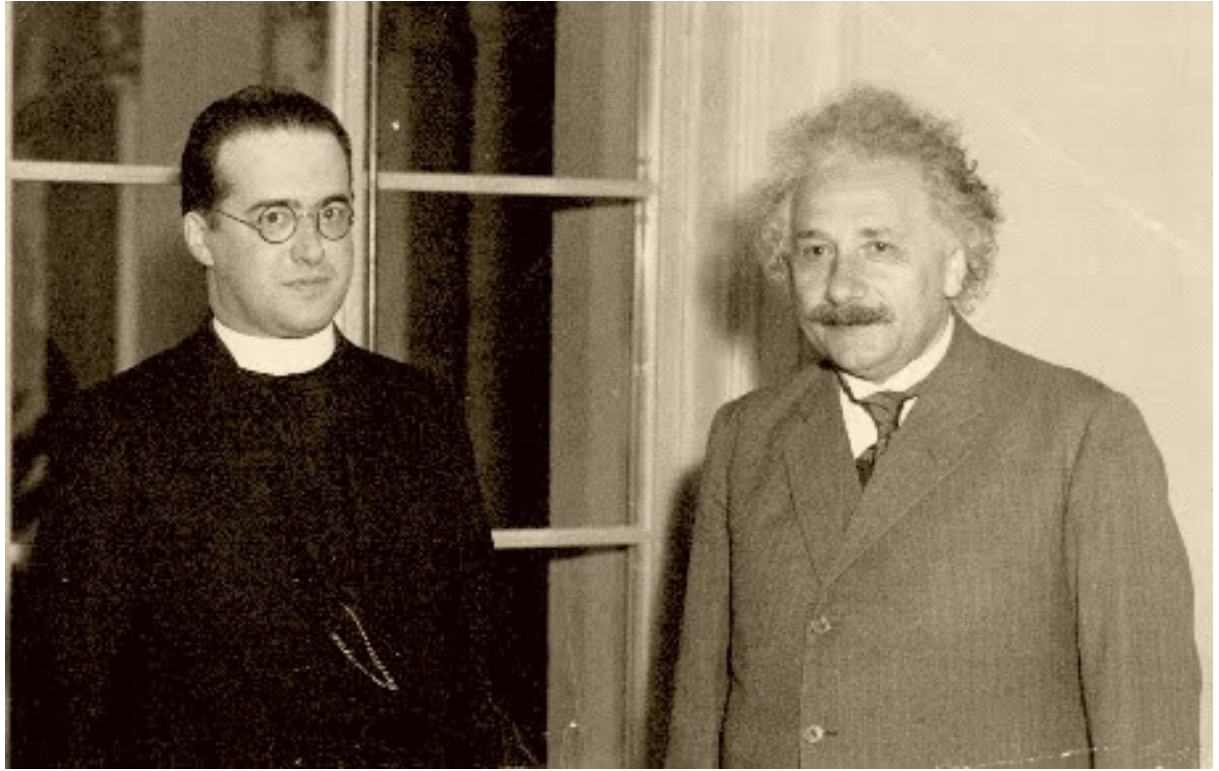
Albert Einstein and Kurt Gödel

Gödel constructed a rotating universe model as a solution of Einstein's field equations



Einstein and De Sitter 1932

They constructed the so called Einstein-de Sitter universe model, a model with Euclidean spatial geometry filled by cold, pressure free matter.

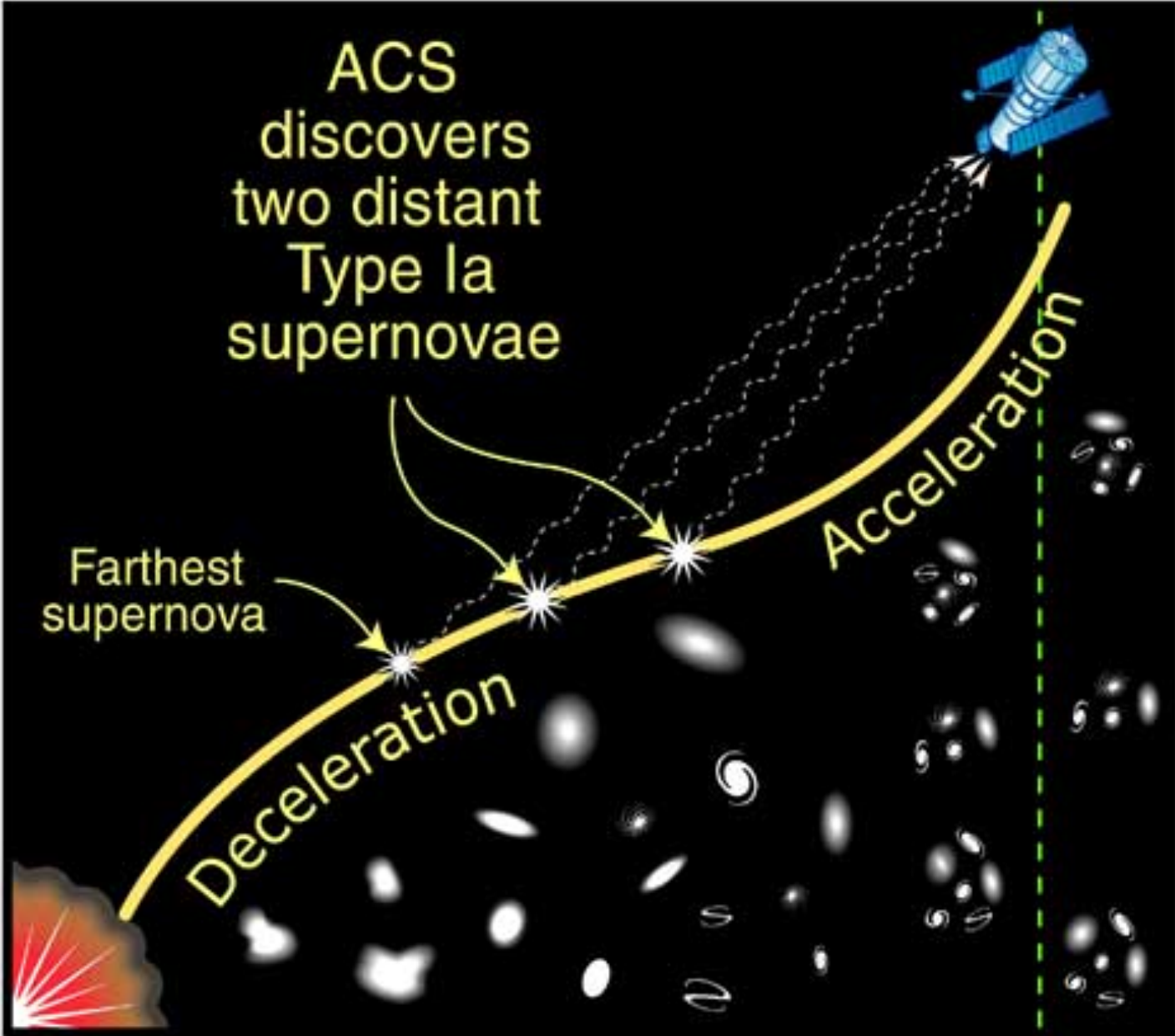


Alexander Friedmann 1888-1925

Georges Lemaître 1894-1966

with Einstein

Expansion of universe



ACS
discovers
two distant
Type Ia
supernovae

Farthest
supernova

Acceleration

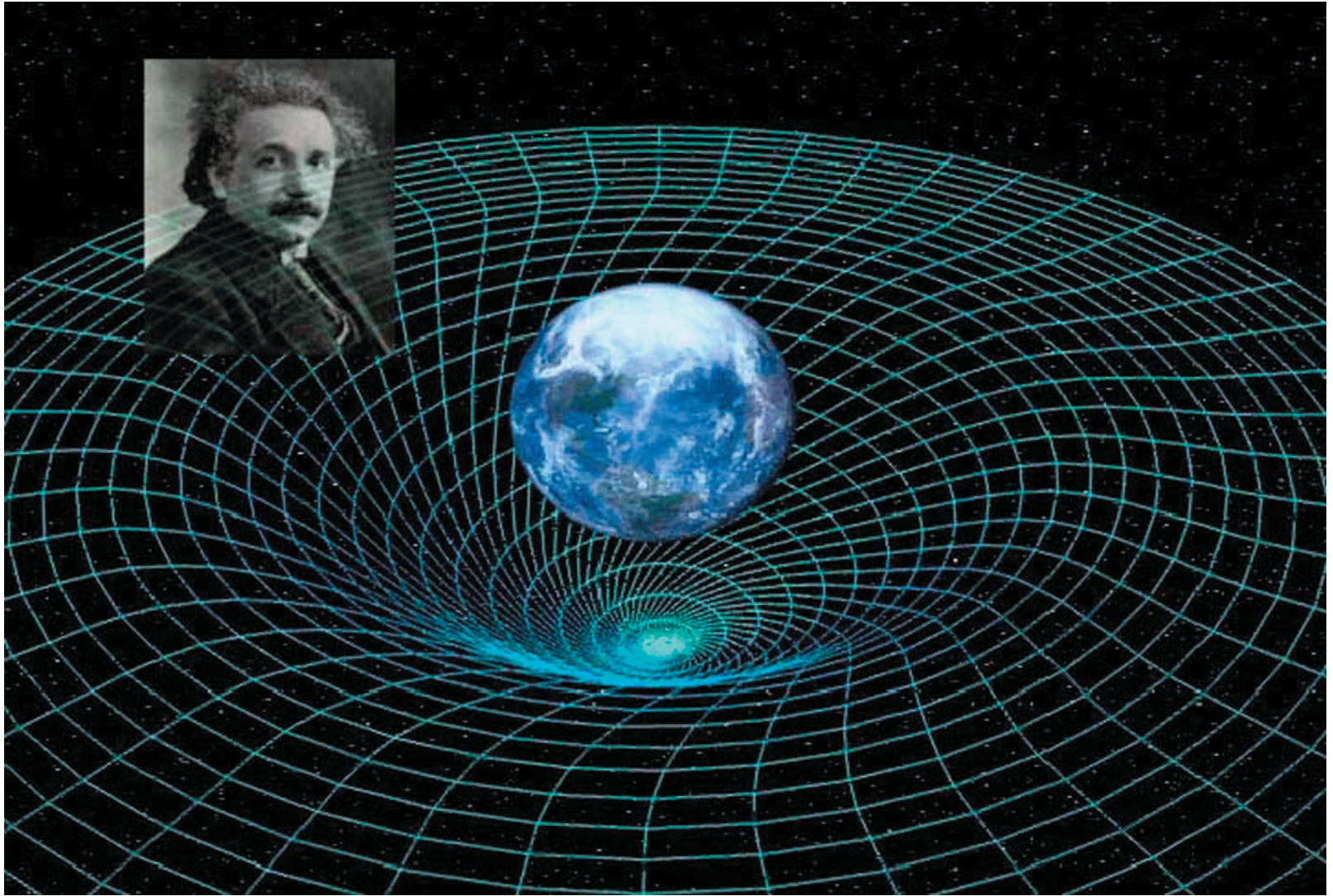
Deceleration

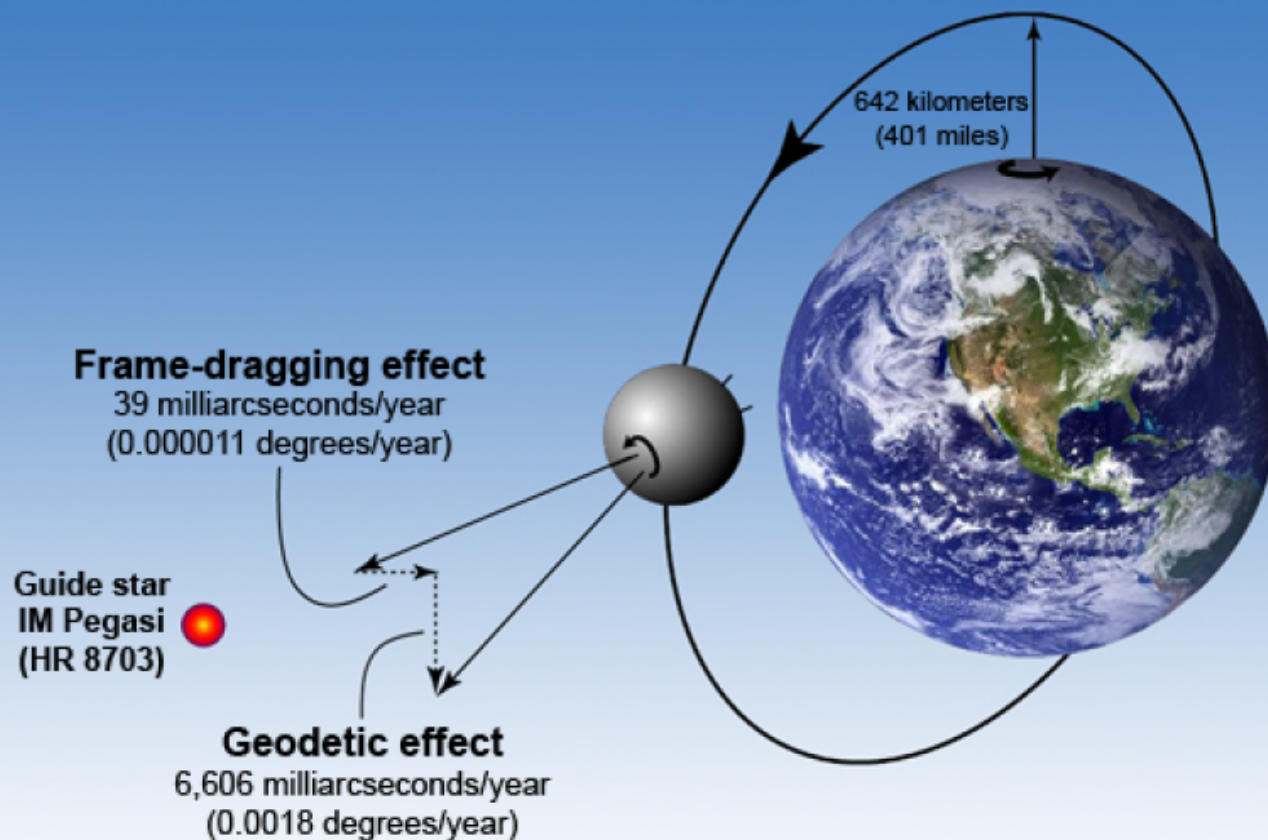
Big
Bang

10 billion
years
ago

5 billion
years
ago

Today





	Measured	Predicted
Geodetic precession (mas)	6602 ± 18	6606
Frame-dragging (mas)	37.2 ± 7.2	39.2



<https://delorian64.wordpress.com/page/5/>

***The Norwegian Academy of Science and Letters awards the
2016 Kavli Prize in Astrophysics to:***



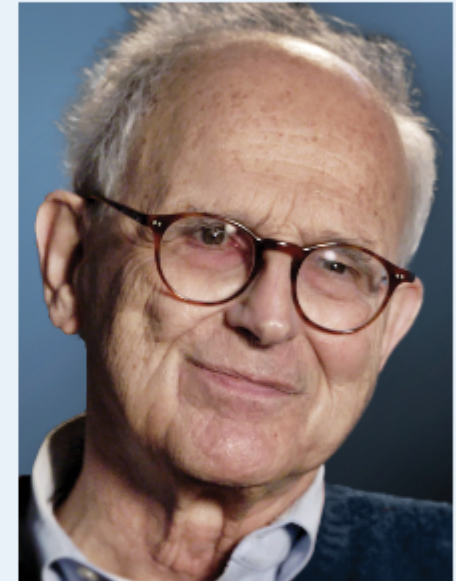
Ronald Drever

*California Institute of Technology,
USA*



Kip Thorne

*California Institute of Technology,
USA*

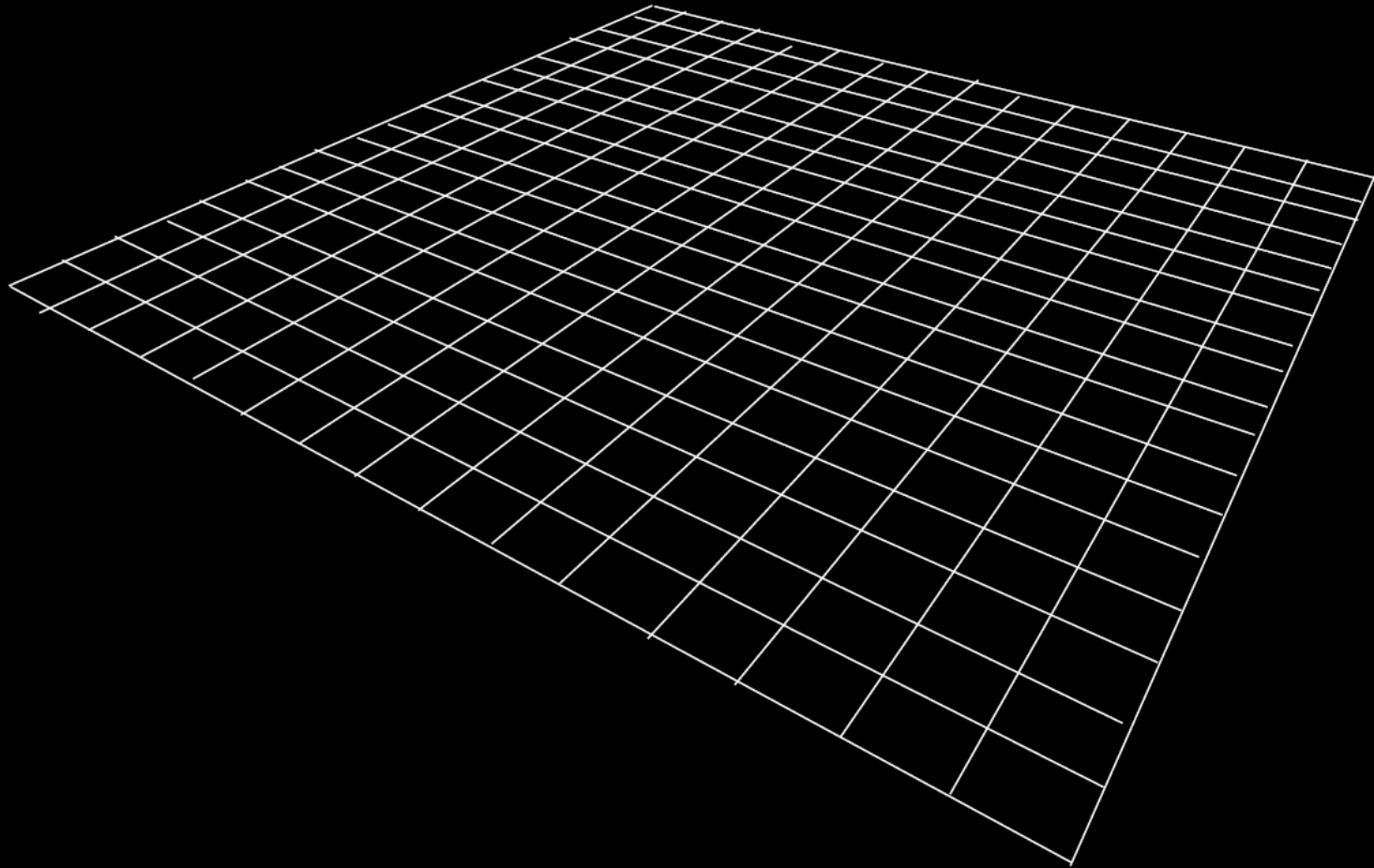


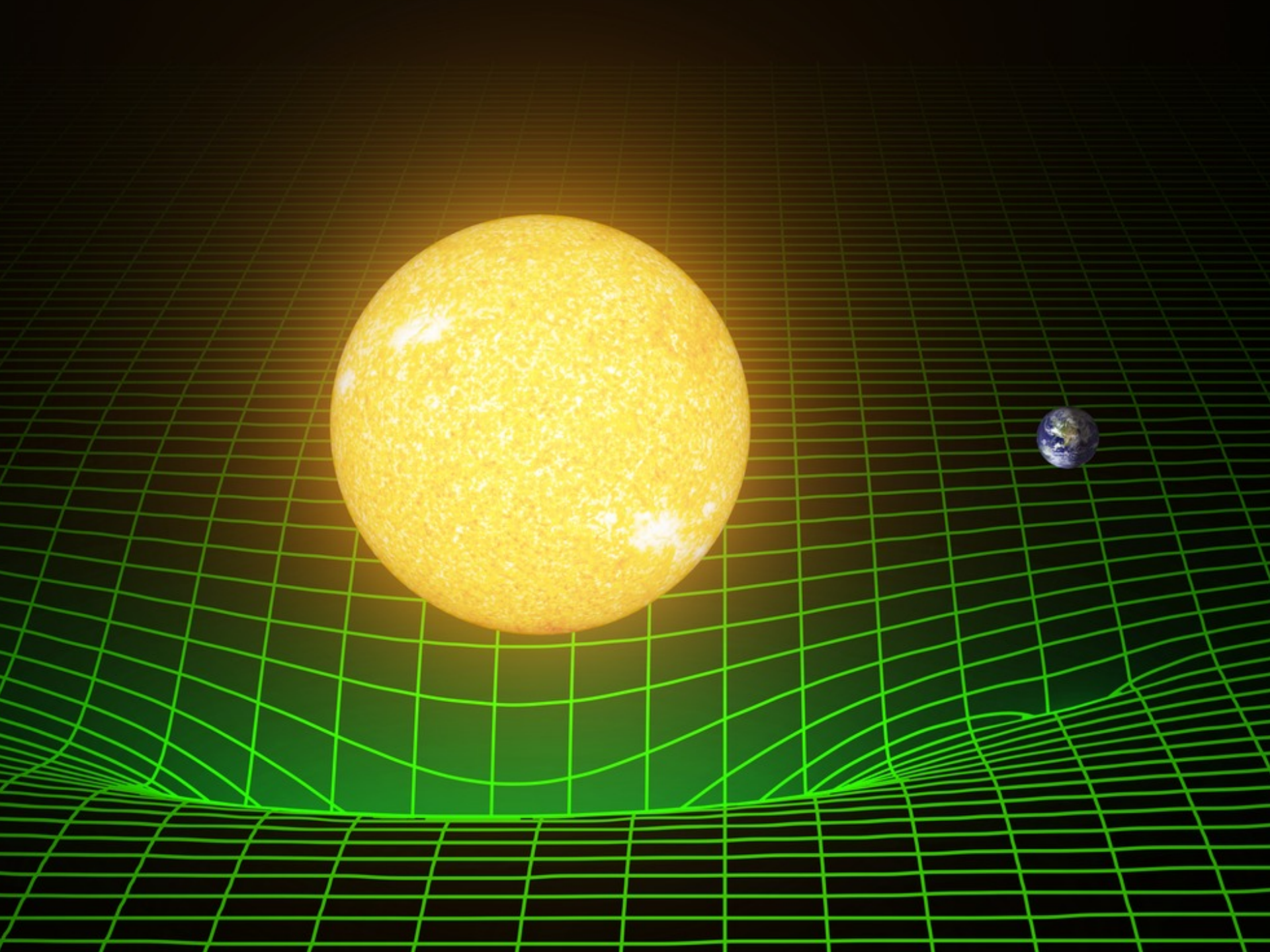
Rainer Weiss

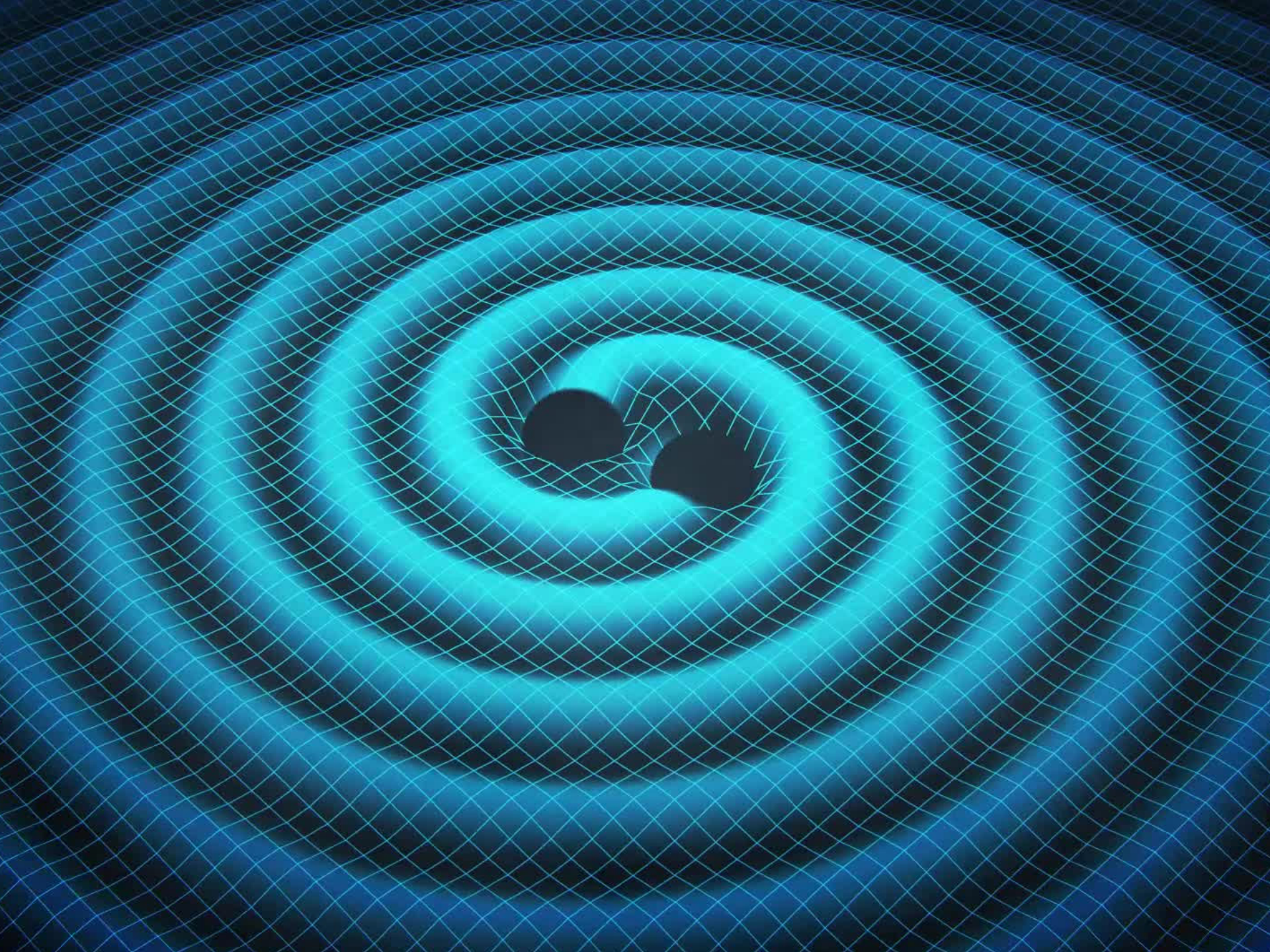
*Massachusetts Institute of
Technology, USA*

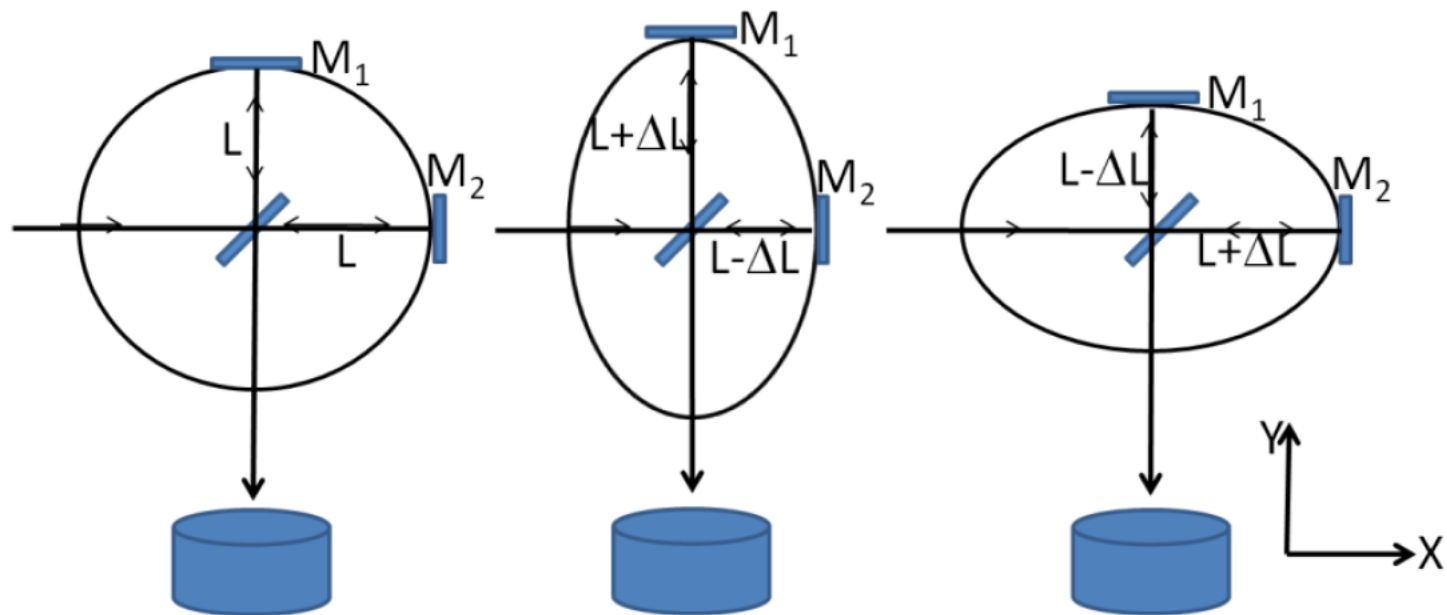
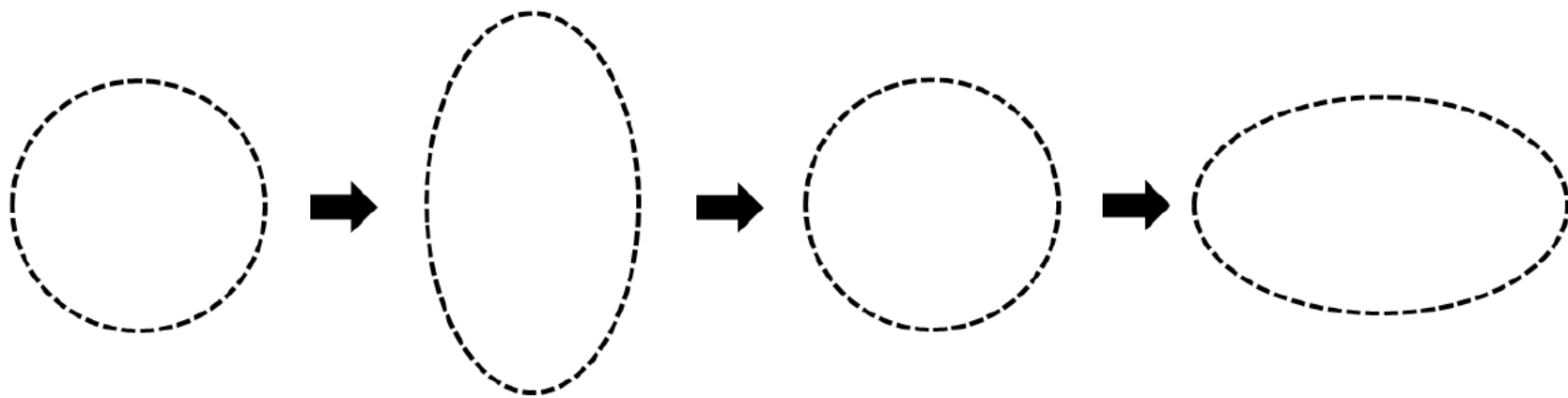
“for the direct detection of gravitational waves.”

Mass-free “flat” Space-time

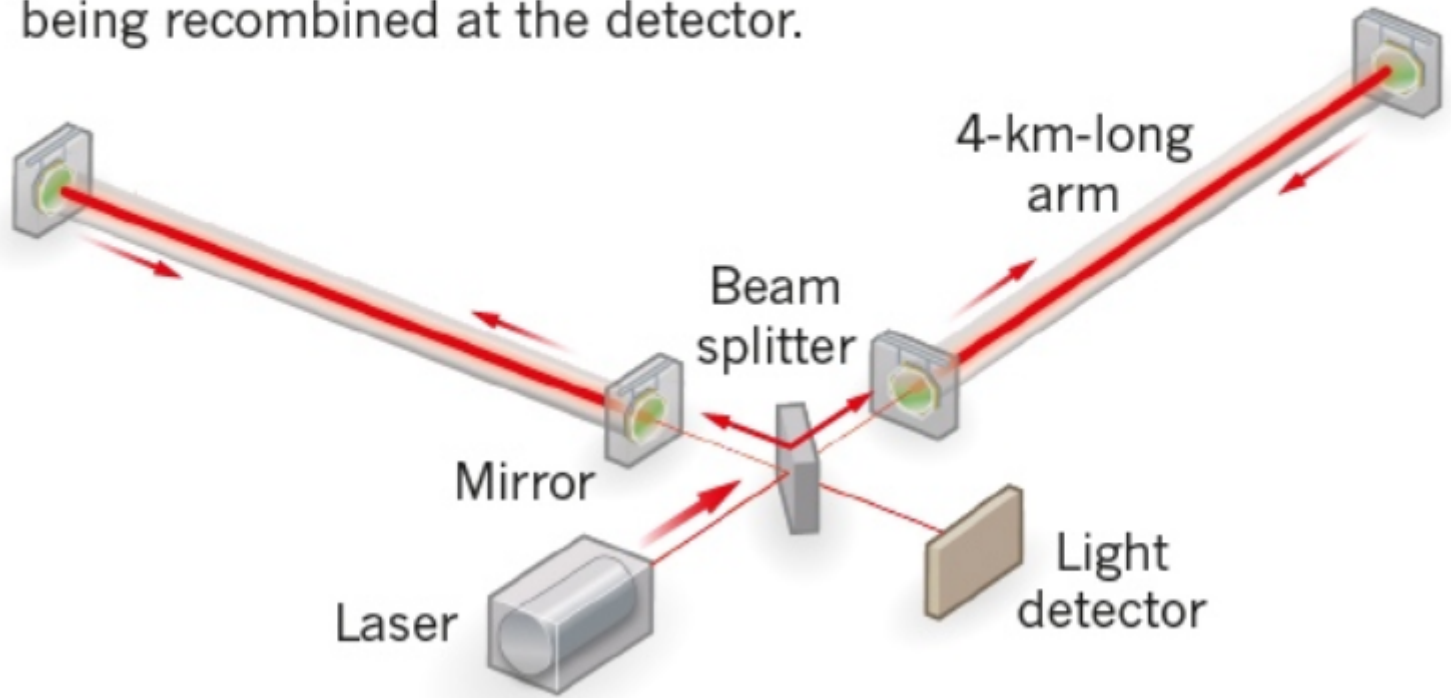




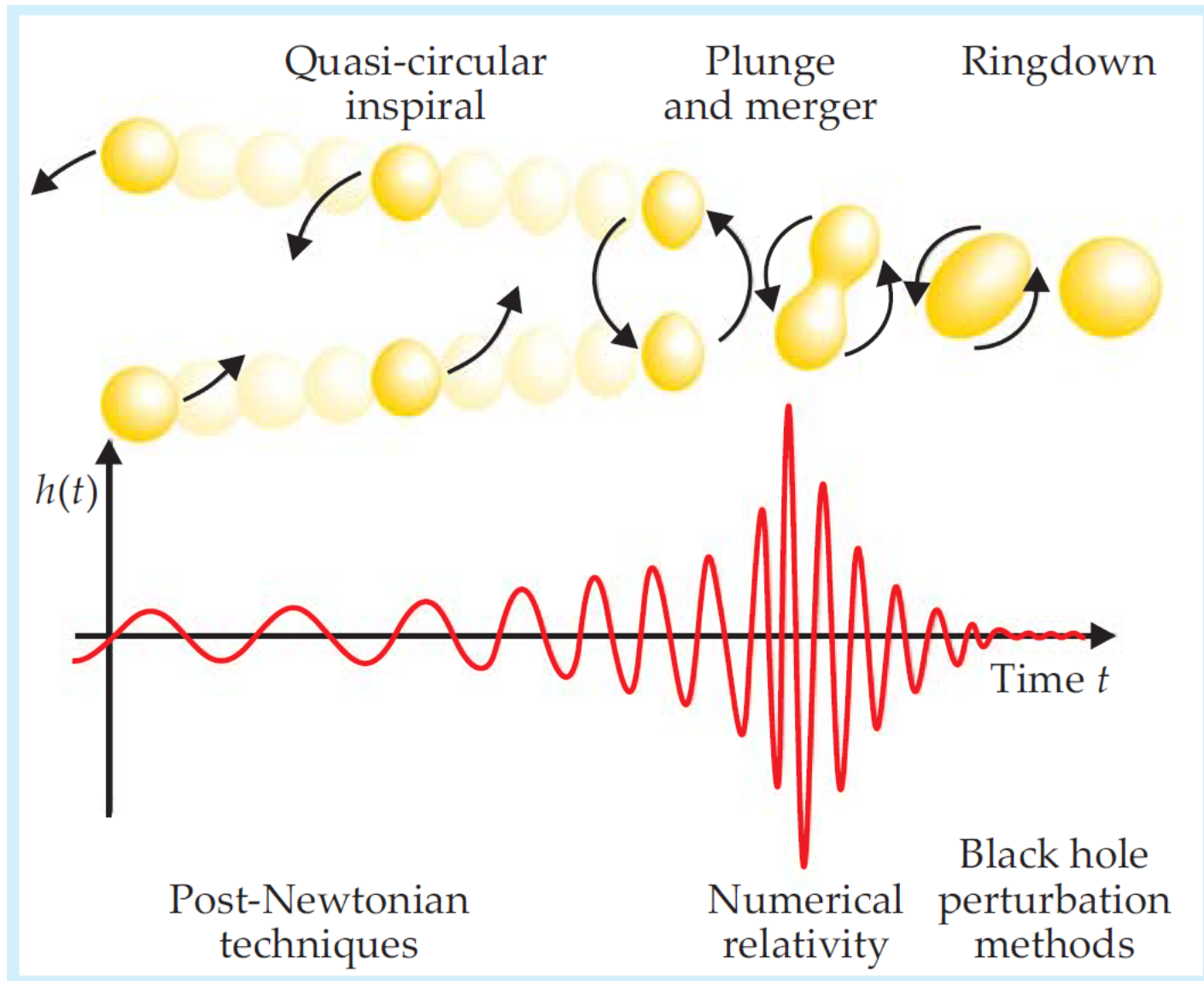




In the LIGO facility, a laser beam is split to travel down two perpendicular 4-kilometre tunnels. The beams then reflect back and forth before being recombined at the detector.



Binary black hole mergers in Physics Today October 2011





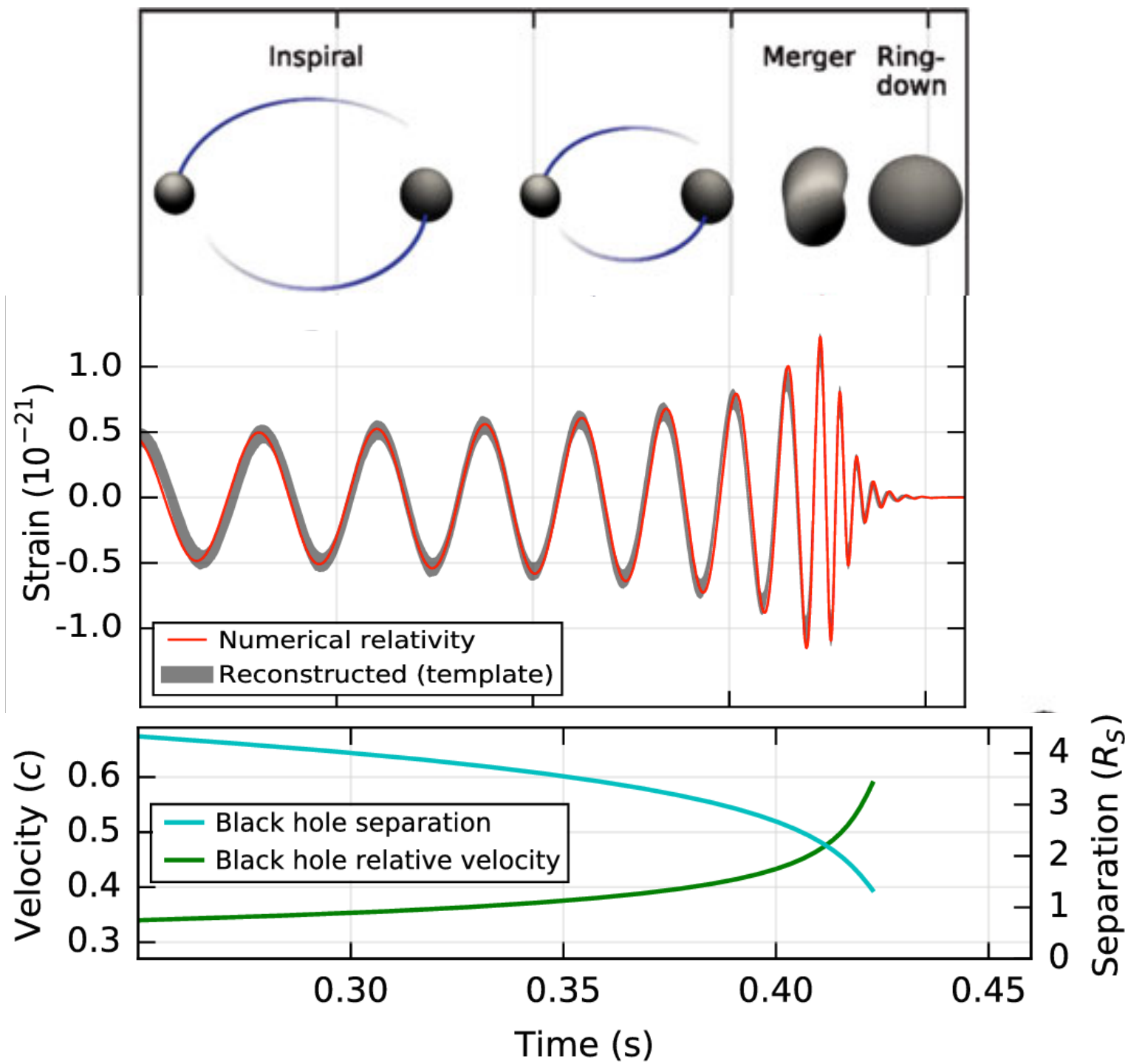
Observation of Gravitational Waves from a Binary Black Hole Merger

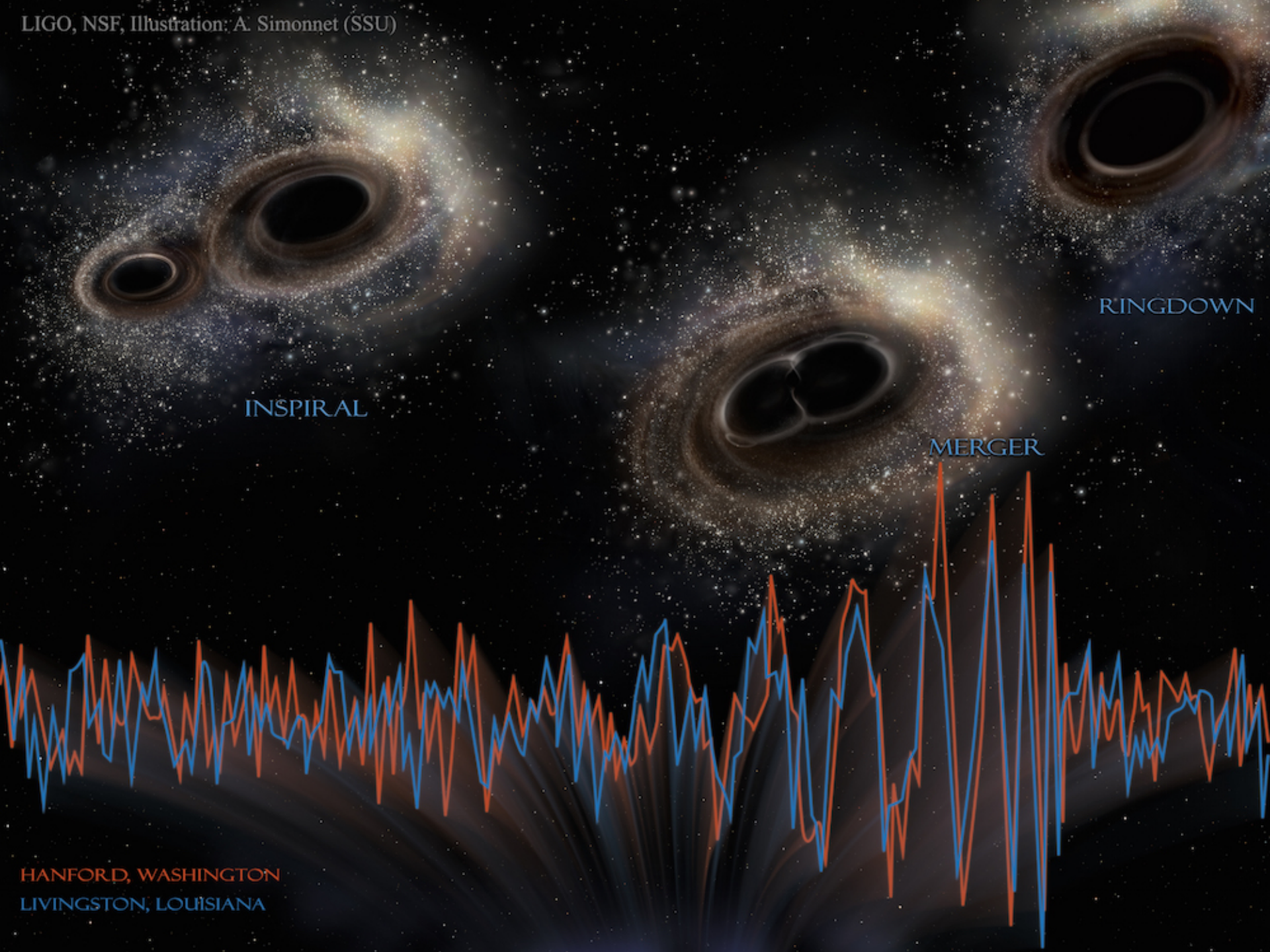
B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

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On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21} . It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ . The source lies at a luminosity distance of 410_{-180}^{+160} Mpc corresponding to a redshift $z = 0.09_{-0.04}^{+0.03}$. In the source frame, the initial black hole masses are $36_{-4}^{+5}M_{\odot}$ and $29_{-4}^{+4}M_{\odot}$, and the final black hole mass is $62_{-4}^{+4}M_{\odot}$, with $3.0_{-0.5}^{+0.5}M_{\odot}c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.





INSPIRAL

RINGDOWN

MERGER

HANFORD, WASHINGTON
LIVINGSTON, LOUISIANA

