

QUATERNIONS WITH APPLICATIONS

The complex numbers do not have any *proper* counterpart in three dimensional space, but in four dimensions one can do something. The *quaternions* are quantities

$$q = x + y\mathbf{i} + u\mathbf{j} + v\mathbf{k}$$

in the four dimensional space \mathbb{R}^4 . In some sense they are a substitute for the complex numbers, but they do not commute under multiplication: the *order* of the factors count. Though living in 4 dimensions, they have a lot of applications to “our physical space” \mathbb{R}^3 : “gimbal lock”, navigation, flight dynamics, robotics, and so on. Even Maxwell’s Equations can be written with quaternions.

In particular, rotations in \mathbb{R}^3 can be conveniently described by quaternions. Thus the calculations are in 4 dimensions, while the result is read off in only 3 dimensions, one coordinate being left as “meaningless”.

The project would be to first learn the rules for the calculations and then to study some interesting application(s), for example a simple way to represent rotations in space. A lot of information is on internet. It is required that the student knows at least how to calculate with ordinary complex numbers. Some basic knowledge of vectors and matrices is expected. No infinitesimal calculus is needed, but patience with explicit calculations is an advantage.

This can be useful for students in Physics or Ingeneering.

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