NORMALIZATIONS, CONSTANTS, AND FUNDAMENTAL SOLUTIONS

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Some parabolic equations like the Evolutionary p-Laplace Equation

$$\frac{\partial u}{\partial t} \;=\; \nabla\cdot\big(|\nabla u|^{p-2}\nabla u\big), \qquad 2<p,\infty,$$

have explicit "fundamental solutions": in this case the celebrated Barenblatt solution

$$\mathfrak{B}(x,t)=\begin{cases} t^{-\frac{n}{\lambda}}\left[C-\frac{p-2}{p}\lambda^{\frac{1}{1-p}}\left(\frac{|x|}{t^{1/\lambda}}\right)^{\frac{p}{p-1}}\right]_+^{\frac{p-1}{p-2}}, & \text{when} \quad t>0 \\ 0, \quad \text{when} \quad t\le 0 \end{cases} \tag{1}$$

found in 1951, $\lambda=n(p-2)+p$ and $x=(x_1,x_2,\dots,x_n)$. Notice the constant C. It is a solution of

$$\frac{\partial \mathfrak{B}}{\partial t}-\nabla\cdot(|\nabla\mathfrak{B}|^{p-2}\nabla\mathfrak{B})=c\,\delta$$

in the sense of distributions (δ = Dirac's delta). The constant C in the solution has to be chosen so that $c=1$ in front of Dirac's delta. In literature various values are given that do *not* agree! Misprints and errors! Neither is the case $p<2$ properly treated. Thus it would be useful to have it all calculated and checked. There are similar problems with some other related equations. In some cases formulas are presently available only in one (space) dimension and have to be amended.

The task is to calculate the constants, to verify that the solutions are correct, and, perhaps, to make some pictures. The calculations are long, done by hand or with Maple, and require patience. The Gamma and Beta functions will appear (but they are easy to learn). This is suitable for someone who likes explicit calculations. Knowledge of calculus is absolutely required.