MEANS OF SOME ARITHMETIC FUNCTIONS

Advisors: Andriy Bondarenko and Kristian Seip

Problem: Let n be a positive integer. Denote by $\Omega(n)$ the number of prime divisors of n counting multiplicities (for example $\Omega(12) = 3$). We propose to estimate the size of

$$\sum_{n=1}^{N} e^{r\Omega(n)}$$

for large N when r is a fixed real number. This problem is closely related to the following classical result: Let $d(n)$ be the number of *all* divisors of the positive integer n. Then for a fixed $r \in \mathbb{R}$

$$\sum_{n=1}^{N} d(n)^r \approx N(\log N)^{2^r-1}$$

for large N. This relation can be seen immediately from the identity $2^{\Omega(n)} = d(n)$ which holds for all squarefree numbers n. The latter result can be proven using the Riemann zeta function. However, for $r = 1$ it follows easily from the beautiful identity

$$\sum_{n=1}^{N} d(n) = \sum_{m=1}^{N} \left[\frac{N}{m}\right],$$

where $[x]$ stands for the integer part of x.

The problem is suitable for someone with an interest in number theory.