

PROJECT: CURVATURE OF ZERO CURVES OF HARMONIC FUNCTIONS

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Motivation. The level sets of harmonic functions and eigenfunctions of the Laplace-Beltrami operator attract a lot of attention of pure and applied mathematicians. Recently, new results appeared and the interest is increasing. At the same time some elementary questions remain open. In this project a question of the curvature of the zero set of two dimensional harmonic function will be studied.

Problem. Let h be a harmonic function in the unit circle such that its zero curve passes through the origin and intersects the circle only at two points. Kuran showed that the curvature of the zero curve at the origin is bounded from above by 8. Assume that we fix the direction of the tangent line l to the zero curve at the origin. Using the ideas of Kuran, we want to show that the zero curve never intersects the disk of radius $1/8$ tangent to l at the origin. Further, the problem is to find the largest domain free of zeros and construct some examples of such harmonic functions, trying to minimize the zero-free domain.

Details. This problem was recently studied by Steinerberger, he recovered the result of Kuran and gave an example that shows the sharpness of the curvature constrain. Analysis of Kuran's prove shows also that one can prove more than just curvature estimate and there is some guaranteed zero-free domain. The aim of the project is to understand the details of the proof and try to describe the largest domain inscribed in any nodal set.

Prerequisites and workload. A good understanding of basic courses including Mathematics 4K is required. Some experience with MATLAB would be an advantage. Other necessary training will be provided. The expected workload is about 80 hours.