STUDFORSK PROJECT: THE PARTIAL SUM OPERATOR ON L^p AND H^p

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If f is an integrable function on the unit circle $\mathbb{T}$ (i.e., it belongs to $L^1(\mathbb{T})$), then it has a Fourier series

$$f(t) \sim \sum_{k=-\infty}^{\infty} \hat{f}(k) e^{2\pi i k t}, \quad \hat{f}(k) := \int_0^1 f(t) e^{-2\pi k t} dt.$$

It is a delicate question in which sense the Fourier series represents f, depending on the properties of f. In this project, we will be concerned with a simpler object, namely the classical partial sum operator S_N defined by the

$$S_N f(t) := \sum_{|k| \le N} \hat{f}(k) e^{2\pi i k t}.$$

This much studied operator can be represented as an integral operator obtained by convolving f with the Dirichlet kernel of length N. If we replace $L^p(\mathbb{T})$ by $H^p(\mathbb{T})$, then we may make sense of S_N also for $0 < p < 1$, if we replace Fourier series by Taylor series by which functions in $H^p(\mathbb{T})$ are represented in the unit disc $|z| < 1$.

It is clear that S_N is a bounded operator on $L^p(\mathbb{T})$ for $1 \le p < \infty$ and on $H^p(\mathbb{T})$ for $0 < p \le \infty$, and it is well known that we have uniform boundedness when $1 < p < \infty$. The student is supposed to give an overview of what is known about the precise value of the norm of S_N when acting on these spaces and if possible provide examples and/or proofs to improve on what is found in the literature. A possible follow-up problem is to consider multi-dimensional analogues of S_N.