Prosjekt: Manifolds of mappings and the group of bisections of a Lie groupoid
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Bakgrunn: It is well known that one can associate to each (Lie) groupoid a group of its bisections. In [SW15a] we have shown that this group can be turned into an (infinite-dimensional) locally convex Lie group in the case of a groupoid with compact manifold of objects. Contrary to older results (see [Ryb02]), this allows one to use mapping space techniques to identify the Lie algebra (together with a suitable topology). Subsequently these results have been exploited in [SW15b] to connect the geometry of infinite-dimensional Lie groups and Lie groupoids.

However, the techniques used in [SW15a] required the compactness assumption as a crucial ingredient. As there are many interesting examples of bisection groups of groupoids (over non-compact base) a generalisation of the above results is highly desirable. The main problem one has to circumvent is that the topology of mapping spaces (with non-compact source manifold) is significantly more complicated than in the compact case. Taking an alternative description of the function space topologies, it should be possible to adapt techniques from the literature to deal with the new situation.

Problem: Construct a suitable topology on $C^\infty(M, N)$ where M is a (non-compact) finite-dimensional manifold and N is a locally convex manifold. This topology should generalise the so called (FD)-topology on $C^\infty(M, N)$ from [Mic80] (where M, N are finite-dimensional) to the infinite-dimensional setting. In an (optional) second step use these constructions to obtain manifold structures on mapping spaces.

Spesifikasion: The project consists of the following tasks

(a) Construct the topology on $C^\infty(M, N)$ for N not necessarily finite-dimensional and M non-compact,

(b) Study the topology on $C^\infty(M, N)$. In particular, show for N finite-dimensional that one recovers the (FD)-topology,

(c) Establish continuity for certain composition and push-forward maps with respect to the topology from (a),

(d) Construct the manifold structure on $C^\infty(M, N)$ turning it into a locally convex manifold modelled on spaces of compactly supported sections,

(e) Show that the group of bisection $\mathrm{Bis}(\mathcal{G})$ of a a Lie groupoid $\mathcal{G} = (G \rightrightarrows M)$ turns (under certain assumptions) into a submanifold of $C^\infty(M, G)$ and this structure turns the group into a Lie group.

Item (d)-(e) will not necessarily be part of the project as (a)-(c) are the core of the problem.

Note that if N is a Banach manifold, items (c)-(e) follow easily by arguments analogous to [Mic80] after (a) and (b) have been established.

Forkunnskaper: The student should have successfully completed the basic course on topology and the basic course on manifolds. No knowledge about Lie groups or Lie groupoids is needed.

Opplæring: At the beginning of the project the candidate will receive an introduction to topologies on spaces of (smooth) maps. In particular, this includes an introduction to differential calculus on locally convex spaces.

Tidsramme: The project is estimated to take ca. 100 hours of work (depending also on the number of items from the above list which are included in the project)

References

[Mic80] Michor, P. W. *Manifolds of Differentiable Mappings*, *Shiva Mathematics Series*, vol. 3 (Shiva Publishing Ltd., Nantwich, 1980). URL `http://www.mat.univie.ac.at/~michor/manifolds_of_differentiable_mappings.pdf`

[Ryb02] Rybicki, T. *A Lie group structure on strict groups.* Publ. Math. Debrecen **61** (2002)(3-4):533–548

[SW15a] Schmeding, A. and Wockel, C. *The Lie group of bisections of a Lie groupoid*. Annals of Global Analysis and Geometry **48** (2015)(1):87–123. `doi:10.1007/s10455-015-9459-z`. `arXiv:1409.1428`

[SW15b] Schmeding, A. and Wockel, C. *(Re)constructing Lie groupoids from their bisections and applications to prequantisation* 2015. `arXiv:1506.05415`