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# The Value of Seismic Amplitude Information

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**ABSTRACT**

Gathering the right kind and amount of information is crucial in any decision making process. Estimating the value of information (VOI) is therefore important for acquiring the right kind of information. We present a method for valuing information in the context of spatial decision-making relevant to reservoir development. Our model is applied to decisions about data acquisition in the case of spatially correlated porosity and saturation variables along top reservoir. We illustrate our method with a simple example based on valuing seismic AVO data for decisions regarding well locations.

## INTRODUCTION

When a prospect has been determined from prior knowledge and some tests, one naturally asks questions like: Should we drill here? Do we require more data before we make a decision about drilling? Such questions naturally fit into the notion of value of information (VOI). In this paper we use VOI to evaluate the monetary amount one should be willing to pay for a certain type of data, in situations that naturally exhibit spatial dependence. Our VOI tool is beneficial for decision making. Consider for instance the case where one has defined a prospect from seismic traveltime data, but must now evaluate the options of purchasing zero-offset reflectivity amplitudes (post-stack seismic data), or both amplitude versus offset (AVO) attributes (pre-stack seismic data), or in some cases controlled source electromagnetic (CSEM) data. For better analysis one of course wants all these data, but in practice this is not always worth the cost.

We assess the VOI in the context of seismic AVO data picked at top-reservoir, and in a model with spatially dependent porosity and saturation. Previous work for valuing geophysical data using VOI calculations, without taking into account geostatistical modeling, have been described in e.g. Bickel et al. (2006) and Houck and Pavlov (2006). In Bhattacharjya et al. (2006) we computed VOI for discrete spatial models.

## THEORY

### Lattice model for reservoir variables and seismic data

Suppose reservoir variables are represented on a regular lateral grid. Let $i = 1, \ldots, n$ be the index of the grid cells. Here $n = n_1 n_2$ and $(n_1, n_2)$ are the number of grid cells in north and east directions. Grid cell reservoir variables are porosity $\phi_i$ and water saturation $s_i$. We treat the reservoir thickness $h_i$ as fixed, typically determined from seismic traveltime data. Seismic AVO data $\boldsymbol{d}_i$ can be acquired and processed at every grid cell. AVO data contains zero offset reflectivity and AVO gradient.

### Value of information

Suppose that the cost of drilling a well at site $i$ is $C_i = C$ and that the maximum revenue per cell is $R$, which is set to a constant and represents the price of oil per unit volume multiplied with the lateral grid size and assuming a constant recovery factor. The value is related to Net-to-Gross and for one well it is

$$v_i = \max[Rh_i\phi_i(1 - s_i) - C, 0], \tag{1}$$

where we only drill at cell $i$ if the value is larger than 0. The total value of the reservoir domain is $v = \sum_{i=1}^{n} v_i$, assuming that each decision is taken marginally. Of course, the values of porosity and saturation are not known exactly and we hence calculate their expected value based on the currently available information. The VOI is defined as the expected gain in value after conditioning on the relevant new information. In our case the prior value is the expected profit without purchasing AVO data, posterior value the expected profit with seismic AVO data, and VOI the difference between these two:

$$\text{VOI} = \sum_{i=1}^{n} E_{\boldsymbol{d}}\left(\max\{Rh_iE[\phi_i(1 - s_i)|\boldsymbol{d}] - C, 0\}\right) - \sum_{i=1}^{n} \max\{Rh_iE[\phi_i(1-s_i)] - C, 0\}. \tag{2}$$

We take the expectation with respect to the seismic data $\boldsymbol{d}$ in the first term of equation (2), since we want to make the *decision* before the data is actually purchased. Note that the VOI is independent of the cost of the data. If the VOI is larger than the cost of purchasing AVO data, we *decide* to buy these data.

**Prior model for saturation and porosity**

We represent water saturation by $s_i \in (0.1, 0.9)$. Similarly for porosity; $\phi_i \in (0.15, 0.4)$. We obtain variables on the real line by logistic transforms:

$$m_{s,i} = \log\left(\frac{s_i - 0.1}{0.9 - s_i}\right), \quad m_{\phi,i} = \log\left(\frac{\phi_i - 0.15}{0.4 - \phi_i}\right) \quad \text{for all } i. \tag{3}$$

Let $s_i = s_i(m_{s,i})$ and $\phi_i = \phi_i(m_{\phi,i})$ generally denote the inverse logistic transform of equation (3). We assign prior pdfs to the logistic counterparts by computationally efficient Gaussian Markov random fields, see Rue and Held (2005). The prior for $\boldsymbol{m} = (m_{s,1}, m_{\phi,1}, m_{s,2}, \dots, m_{\phi,n})$ is then $p(\boldsymbol{m}) = N(\boldsymbol{m}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu}$ is a fixed size $2n$ mean vector based on prior knowledge about saturation and porosity, typically represented by only two parameters $\mu_s$ and $\mu_\phi$ constant across all cells. Further, the covariance matrix $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_0 \otimes \boldsymbol{Q}^{-1}$ is composed of the marginal cellwise $2 \times 2$ covariance matrix $\Sigma_0$ which is diagonal with entries $\sigma_s^2$ and $\sigma_\phi^2$ if the two variables are considered independent a priori, and a spatial size $n \times n$ correlation matrix $\boldsymbol{Q}^{-1}$. Matrix $\boldsymbol{Q}$ is specified via a nearest neighbor non-zero structure defined by 1 on the diagonal, $q$ at entries $(i, j)$ where cells $i$ and $j$ are neighbors on the lattice, and 0 otherwise. Hence, if $q = 0$, cells are treated independently, whereas $q < 0$ imposes positive spatial correlation.

The prior expectation $E[(1 - s_i)\phi_i]$ in equation (2) is computed by linearizing the logistic equations for $s_i$ and $\phi_i$.

**Likelihood of AVO data**

Saturation and porosity are linked to elastic moduli and seismic velocities by rock physics relations, Mavko et al. (1998). These are further related to seismic AVO data through the Aki and Richards approximation of the Zoeppritz equations (Mavko et al. (1998)), assuming constant cap rock properties. Dependence of the bulk modulus on saturation is through the usual Gassmann relations. Shear modulus remains independent of saturation, while density depends linearly on saturation. Both the moduli, and the density vary with porosity, and in general the moduli-porosity relation should be calibrated from well logs. Here we use a linearized relation based on data from Bachrach (2006), representing shaley sands.

Altogether, we can compute $E(\boldsymbol{d}_i|\boldsymbol{m}_i) = \boldsymbol{f}_i(\boldsymbol{m}_i)$, $i = 1, \dots, n$, where $\boldsymbol{d}_i = (R_{0,i}, G_i)$, indicating zero-offset reflectivity and AVO gradient. The nonlinear function $\boldsymbol{f}_i(\boldsymbol{m}_i)$ ties saturation and porosity to these seismic attributes. Figure 1(left) shows 500 realizations where $\boldsymbol{m}_i$'s are drawn from the prior model and taken through the forward model $\boldsymbol{f}_i(\boldsymbol{m}_i)$. The likelihood pdf of $\boldsymbol{d} = (\boldsymbol{d}_1, \dots, \boldsymbol{d}_n)$ is $p(\boldsymbol{d}|\boldsymbol{m}) = N(\boldsymbol{f}(\boldsymbol{m}), \boldsymbol{T})$, where $\boldsymbol{f} = [\boldsymbol{f}_1(\boldsymbol{m}_1), \dots, \boldsymbol{f}_n(\boldsymbol{m}_n)]$, and $\boldsymbol{T}$ a $2n \times 2n$ block diagonal matrix consisting of the AVO standard deviations $\sigma_{R_0}$ and $\sigma_G$ on the diagonal, and fixed correlation coefficient set to $-0.7$ here.

**Posterior**

For the simple $n = 1$ case we show the posterior for $(s_1, \phi_1)$ when $\mu_s = \mu_\phi = 0$, $\sigma_s = \sigma_\phi = 1$, $\sigma_{R_0} = 0.05$ and $\sigma_G = 0.15$ in Figure 1(middle). Here, we condition on data $\boldsymbol{d}_i = (0.05, -0.05)$. In Figure 2(right) we plot this posterior for the case with 1/2 as large likelihood standard deviations. Reservoir variables are better determined, showing the added certainty induced by for instance expensive processing of data. However, it is difficult to guess from these posteriors the *value* of the higher quality data. This is the major advantage with VOI which provides this amount in monetary units.

For the general multidimensional case, let $\boldsymbol{F}$ denote the Jacobian of the forward

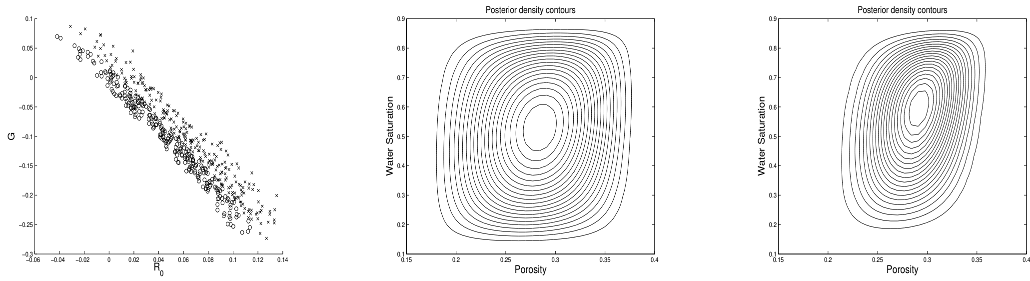


Figure 1: Left) AVO data $R_0$ and $G$ from forward model. Circles: small water saturation; crosses: large water saturation. Middle) Posterior solution based on only $(R_0, G) = (0.05, -0.05)$ data. Right) Posterior solution based on the same data, but with half the standard deviation in the likelihood noise term.

model $\boldsymbol{f}$ at linearization point $\boldsymbol{\mu}^\star$. The posterior mean is

$$E(\boldsymbol{m}|\boldsymbol{d}) = (\boldsymbol{\Sigma}^{-1} + \boldsymbol{F}'\boldsymbol{T}^{-1}\boldsymbol{F})^{-1}(\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{F}'\boldsymbol{T}^{-1}\boldsymbol{z}), \qquad \boldsymbol{z} = \boldsymbol{d} - \boldsymbol{f}(\boldsymbol{\mu}^\star) - \boldsymbol{F}\boldsymbol{\mu}^\star. \quad (4)$$

The linearization process is iterated until reaching the posterior mode; argmax$[p(\boldsymbol{d}|\boldsymbol{m})p(\boldsymbol{m})]$. The posterior covariance is $(\boldsymbol{\Sigma}^{-1} + \boldsymbol{F}'\boldsymbol{T}^{-1}\boldsymbol{F})^{-1}$. The expectation $E[(1-s_i)\phi_i|\boldsymbol{d}]$ is computed from the marginals of this Gaussian approximation. To get the posterior value in equation (2) we must also take the expectation over $\boldsymbol{d}$ using Monte Carlo integration.

**Pseudoalgorithm**

- Estimate prior cellwise value $v_i^{\text{prior}} = \max\{Rh_iE[\phi_i(1-s_i)] - C, 0\}$. Calculate the **prior value** as $v^{\text{prior}} = \sum_i v_i^{\text{prior}}$.
- Repeat the following $b = 1, \dots, B$ times ($B \approx 10000$)
  1. Draw $\boldsymbol{m} = \boldsymbol{m}^b$ from prior, then draw $\boldsymbol{d} = \boldsymbol{d}^b$ from likelihood given $\boldsymbol{m}$. This is a Monte Carlo sample from $p(\boldsymbol{d})$.
  2. Solve equation (3) iteratively to find the posterior mode. Fit a Gaussian pdf at the mode.
  3. Estimate the posterior value for this dataset $v_b^{\text{post}} = \sum_i \max\{Rh_iE[\phi_i(1-s_i)|\boldsymbol{d}] - C, 0\}$.
- Calculate the **posterior value** $v^{\text{post}} = \frac{\sum_{b=1}^{B} v_b^{\text{post}}}{B}$.
- The value of information is VOI $= v^{\text{post}} - v^{\text{prior}}$.

## EXAMPLES

For a simple case with $n = 1$, let parameters be as in the posterior description above (Figure 1). The thickness is $h_i = 20$m. The cost of drilling a well is 2 million \$. Let $R = 300 * 50^2$, where 300\$ refers to the oil price per cubic meter, and $50^2\text{m}^2$ refers to the area of each cell. We consider a change in the likelihood standard deviations $r\sigma_{R_0}$ and $r\sigma_G$, where $r$ is a multiplicative perturbation factor that varies in the interval from 0.4 to 2.2. The results of prior value, posterior value and VOI are plotted in Figure 2. The decrease in VOI is natural as the prior value remains constant at about 0.02 million \$ while the information attained in the posterior goes down. For $r = 1$ VOI is 0.13 million

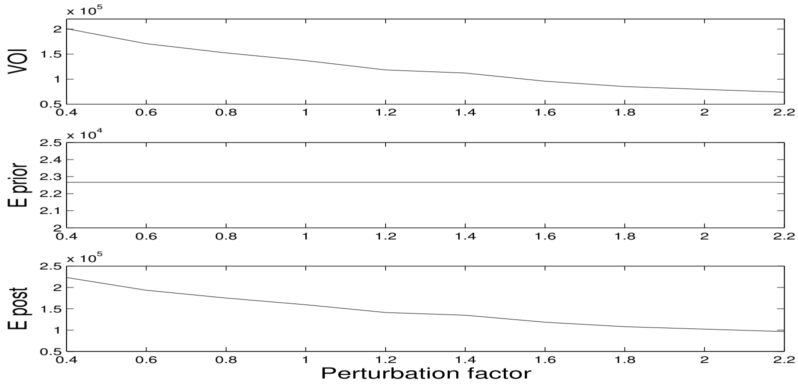


Figure 2: First axis is a perturbation in the likelihood standard deviation. Top) Value of information for different levels of observation noise. Middle) Prior value for different levels of observation noise. Below) Posterior value for different levels of observation noise.

$, while it is 0.19 million $ for $r = 1/2$. If the added processing for high quality $(R_0, G)$ data costs less than this change in VOI, it is worth purchasing the extra effort.

For the spatially correlated case we fix $r = 1$ and regard a grid of size $10 \times 30$. We compare the independent case ($q = 0$) with positive spatial interaction ($q = -0.2$). The special case with independency is identical to $n$ times the one dimensional situation above, and hence VOI $= 300 \cdot 0.13 \approx 39$ million $. The $q = -0.2$ case gives VOI $= 45$ million $. This larger VOI indicates that a positive correlation in the reservoir variables means more valuable seismic AVO data.

## CONCLUSIONS

We have demonstrated a method for computing the Value of Information (VOI) for a spatial model common in petroleum geostatistics. The VOI provides valuable insight for decision making, as it gives the monetary amount that a dataset is worth. We have illustrated our idea with examples on a latent Gaussian field for porosity and saturation and with seismic AVO data.

In a future work we intend to provide larger scale computations of the VOI associated with AVO seismic data and CSEM data.

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