# LITHOLOGY-FLUID INVERSION FROM PRESTACK SEISMIC DATA

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## ABSTRACT

*The focus of the study is on lithology-fluid inversion from prestack seismic data in a 3D reservoir. The inversion is defined in a Bayesian framework where the complete solution is the posterior pdf. The likelihood model relates the lithology-fluid classes to elastic variables and the seismic data, and it follows the lines of Larsen et al. (2006). The prior model for the lithology-fluid characteristics is defined as a profile Markov random field, where allowances to strong lateral couplings between the lithology-fluid classes can be made. The vertical profiles are further defined to follow Markov chain models upward through the reservoir. The likelihood model is approximated, and the corresponding approximate posterior model is given as the complete set of full conditional pdf's for the lithology-fluid classes in the vertical profiles. The approximated posterior model is explored using a block Gibbs simulation algorithm laterally. The profiles are further simulated exactly using the efficient upward-downward algorithm defined in Larsen et al. (2006). The inversion is evaluated on a synthetic 2D reservoir. The lithology-fluid classes in the synthetic reservoir have strong horizontal continuity with thin layers of shale, and the fully coupled 3D model provides reliable results.*

## INTRODUCTION

Prediction and simulation of lithology-fluid (LF) characteristics is important for development of petroleum reserves. The LF characteristics are normally predicted from geological understanding of the reservoir combined with well observations and seismic data. The LF classification problem is ill-posed, as several configurations of LF classes may produce the same seismic data. The focus of the study is on classification of LF classes in a 3D reservoir or general target zone from prestack seismic AVO data. The study draws heavily on results in Buland and Omre (2003) and Larsen et al. (2006). In Buland and Omre (2003) a Bayesian seismic AVO inversion method for elastic parameters was defined. In Larsen et al.

(2006) the inversion was expanded into discrete LF classes along a vertical profile. The major contribution of the current study is on expanding the model into 3D, where lateral continuity of the lithologies is included in the model. We have chosen to define the prior model for the LF classes as a Markov random field, such that spatial couplings are a part of the prior knowledge. This approach appears to be in accordance with the geologists understanding of the problem.

## NOTATION

The objective of the study is to map lithology-fluid (LF) classes in a reservoir. The LF classes are denoted $\pi : \{\pi_{x,t}; (x,t) \in \mathscr{L}_{\mathscr{D}}\}$ where $\mathscr{L}_{\mathscr{D}}$ is a discretization of the reservoir in lateral positions $x \in \mathscr{L}_{\mathscr{D}}^{x}$ corresponding to inline and xline positions, and in time $t \in \{1,\ldots,T\} \in \mathscr{L}_{\mathscr{D}}^{t}$ downward. The inversion is performed from seismic prestack data $d$ for a set of reflection angles. In order to link the LF classes and the seismic data, the elastic parameters P-wave velocity, S-wave velocity and density are used. The logarithm of these elastic parameters is denoted $m$.

## STOCHASTIC MODEL

The inversion problem is defined in a Bayesian setting where the complete solution is the posterior probability density function (pdf)

$$p(\pi|d) = \text{const} \times p(d|\pi)\, p(\pi),$$

with $p(d|\pi)$ being the likelihood model, $p(\pi)$ the prior model and const a normalizing constant which ensures that the posterior pdf integrates to one. From the posterior pdf, the locationwise most probable LF characteristics solution $\hat{\pi}$ and realizations of $\pi$ are available.

### Likelihood Model

In order to link the seismic data and the LF classes, the likelihood is defined as the integral over the elastic parameters $m$ like in Larsen et al. (2006)

$$p(d|\pi) = \int \ldots \int p(d|m)\, p(m|\pi)\, dm$$

where $p(d|m)$ is a seismic response likelihood function and $p(m|\pi)$ is a rock physics likelihood function. The rock physics likelihood model is defined locationwise, see Larsen et al. (2006), and it can be written as the product

$$p(m|\pi) = \prod_{x} \prod_{t} p(m_{x,t}|\pi_{x,t})$$

with $x$ taken over $\mathscr{L}_{\mathscr{D}}^{x}$ and $t$ over $\mathscr{L}_{\mathscr{D}}^{t}$ when not explicitly expressed.

The seismic response likelihood model is defined by a vertical convolution model like in Buland and Omre (2003) given by

$$d = s + e = Gm + e$$

where $s$ is the seismic signal, $e$ is observation error and $G$ is a modeling matrix. The modeling matrix is further defined

$$G = WAD$$

where $W$ is a convolution matrix containing wavelets for different incident angles, $A$ contains linearized Aki-Richards coefficients and $D$ is a differential matrix giving the contrasts of the elastic parameters in $m$.

If $m$ is approximated by a Gaussian random field $p_*(m)$ and the observation error $e$ is Gaussian, the associated posterior pdf $p_*(m|d)$ is also Gaussian and analytically available. The seismic response likelihood model is then defined like in Larsen et al. (2006)

$$p(d|m) = \text{const} \times \frac{p_*(m|d)}{p_*(m)}$$

where $p_*(m)$ and $p_*(m|d)$ are Gaussian prior and posterior pdf's for linearized Zoeppritz AVO inversion, see Buland and Omre (2003).

**Prior Model**

In the prior model, strong horizontal and vertical coupling between the LF classes is modeled. The horizontal coupling is modeled by defining the reservoir as a profile Markov random field. Under this model formulation, the conditional pdf's of the LF profiles $\pi_x : \{\pi_{x,t}; t \in \mathscr{L}_{\mathscr{D}}^t\}$ given the LF profiles in the rest of the field are only dependent upon the LF profiles in a neighborhood of $x$ in $\mathscr{L}_{\mathscr{D}}^X$. This Markov property is expressed

$$p(\pi_x|\pi_{-x}) = p(\pi_x|\pi_y; y \in \delta(x)); \text{ all } x \in \mathscr{L}_{\mathscr{D}}^X$$

where $\pi_{-x} : \{\pi_y; y \in \mathscr{L}_{\mathscr{D}}^X, y \neq x\}$ and $\delta(x)$ is a fixed neighborhood of $x$ in $\mathscr{L}_{\mathscr{D}}^X$. The vertical couplings are modeled by defining the profiles $\pi_x$ as Markov chain models upwards through the target zone like in Larsen et al. (2006). This Markov chain model is expressed

$$p(\pi_x|\pi_y; y \in \delta(x)) = \prod_t p(\pi_{x,t}|\pi_{x,t+1}, \pi_{y,t}; y \in \delta(x)); \text{ all } x \in \mathscr{L}_{\mathscr{D}}^X$$

with $p(\pi_{x,T}|\pi_{y,T}; y \in \delta(x)) = p(\pi_{x,T}|\pi_{x,T+1}, \pi_{y,T}; y \in \delta(x))$ for notational convenience. It can be shown that given the LF classes immediately above and below in addition to the LF classes in the lateral neighborhood, each node is independent of the LF classes in the rest of the field. Hence $\pi$ is a Markov random field in the traditional sense, see Besag (1974).

### Posterior Model

The posterior pdf is completely determined by the likelihood and prior models, and it is given by

$$p(\pi|d) = \text{const} \times \left[ \int \ldots \int \frac{p_*(m|d)}{p_*(m)} \prod_x \prod_t p(m_{x,t}|\pi_{x,t})\, dm \right] p(\pi).$$

The integral is over all configurations of the three elastic parameters in the field, hence computer demanding. An approximation of the likelihood like in Larsen et al. (2006) is used, where spatial correlations in the pdf's $p_*(m)$ and $p_*(m|d)$ are ignored. The approximate posterior pdf is then given by

$$\tilde{p}(\pi|d) = \text{const} \times \prod_x \prod_t \left[ \int \int \int \frac{p_*(m_{x,t}|d)}{p_*(m_{x,t})} p(m_{x,t}|\pi_{x,t})\, dm_{x,t} \right] p(\pi).$$

The integral is now of dimension three, and numerically tractable. As the prior model follows a Markov random field model and the likelihood model factorizes, the associated approximate conditional posterior pdf's can be written

$$\tilde{p}(\pi_x|\pi_{-x}, d) = \text{const} \times \prod_t l(d|\pi_{x,t})\, p(\pi_{x,t}|\pi_{x,t+1}, \pi_{y,t}; y \in \delta(x));\ \text{all } x \in \mathscr{L}_{\mathscr{D}}^X$$

where $l(d|\pi_{x,t})$ is the integral within the parentheses in the expression above.

### Assessment of Posterior Model

The conditional posterior pdf's follow inhomogeneous Markov chain models upward through the reservoir, hence the efficient upward-downward recursive algorithm used to explore the 1D posterior pdf in Larsen et al. (2006) can be used to simulate the conditional pdf's exactly. As the profile Markov random field is defined by the complete set of conditional pdf's, a block Gibbs sampling algorithm may be used laterally. Note that although the model is defined in 3D, the iterative Gibbs sampling algorithm only operates in 2D. The third dimension is simulated by the extremely fast recursive upward-downward algorithm. The Gibbs algorithm actually used is:

**Simulation Algorithm**
*Initiate*
  *Generate arbitrary* $\pi$
*Iterate*
  *Draw x uniform randomly from* $\mathscr{L}_{\mathscr{D}}^{X}$
  *Generate* $\pi_x$ *from* $\tilde{p}(\pi_x|\pi_{-x}, d)$ *by the upward-downward simulation algorithm*

The algorithm converges such that $\pi$ will be a sample from $\tilde{p}(\pi|d)$.

## EMPIRICAL STUDY

The inversion model is evaluated on a synthetic 2D reference reservoir containing the four LF classes gas-, oil-, brine-saturated sandstone and shale. The reference reservoir is shown in Figure 1. Note that the reservoir contains some thin layers of shale in the range 1-3 ms, which is thinner than what is normally referred to as seismic resolution. We use observations in two well locations, and observe the LF characteristics from the corresponding profiles in the reference reservoir exactly without observation error. The well profiles are kept unchanged in the Gibbs sampler algorithm.

The prior model is constructed to model lateral continuity and vertical ordering of the LF classes. We consider a first order neighborhood in each lateral direction, such that $\delta(x) = (x-1, x+1)$, and by lateral symmetry we obtain 10 transition matrices. These matrices are constructed such that most probability is assigned to the transitions with neighbors identical to the node under consideration, and low probability to all possible transitions. The transition matrix with both lateral neighbors being shale is for example

$$P^{t}_{SH,SH} = \begin{pmatrix} 0.0002 & 0 & 0 & 0.9998 \\ 0.0002 & 0.0002 & 0 & 0.9996 \\ 0.0002 & 0.0002 & 0.0002 & 0.9994 \\ 0.0002 & 0.0002 & 0.0002 & 0.9994 \end{pmatrix}$$

with rows and columns corresponding to gas-, oil-, brine-saturated sandstone and shale, respectively.

The rock physics likelihood model is defined by samples from a rock physics model like in Larsen et al. (2006), see Figure 2. When generating the synthetic data, the average value of the samples corresponding to each LF class is used with added heterogeneity.

The seismic response likelihood model is defined by a vertical convolution model with additive noise. The Aki-Richards coefficient in the convolution model is a function of the incidence angles $\theta = (0, 10, 20, 30, 40)^{\circ}$, and we use a Ricker wavelet with frequency 30 Hz and length 61 ms. The observation error is wavelet colored noise. The synthetic seismic signal is generated profilewise from the convolution model, and observation error is added to the signal to obtain the seismic data with signal-to-noise ratio of two. The seismic data stack is shown in Figure 1.

We initiate the simulation algorithm from four extreme configurations of the LF classes and monitor the portion of the LF classes after each sweep, with one sweep corresponding to one update of all the profiles. The convergence plot is shown in Figure 3. It appears that the simulations have converged after 2000 sweeps, which is defined to be the burn-in period.

### Results and Discussion

The focus of the study is on LF characteristics $\pi$, and the complete solution is defined as the approximate posterior pdf $\tilde{p}(\pi|d)$. We calculate the locationwise most probable prediction from the expression

$$\hat{\pi}_{x,t}; \{\underset{\pi_{x,t}}{\text{argmax}}\, \tilde{p}(\pi_{x,t}|d);\ \text{all}\ (x,t) \in \mathscr{L}_{\mathscr{D}}\}.$$

Figure 4 contains the locationswise most probable prediction $\hat{\pi}$ and the reference LF reservoir. The structure in the prediction and the reference reservoir is mostly the same. The well observations do not stand out, indicating that the prediction is reliable in near-well areas and that information contained in the wells is an integral part of the solution. One of the main challenges of the study is to classify thin layers of shale, and we see that layers down to one ms are identified. This is caused by the spatial coupling in the prior model.

Figure 5 contains three independent realizations of LF characteristics generated from the approximate posterior pdf $\tilde{p}(\pi|d)$. The realizations represent the prediction uncertainty, and they can be considered as possible LF characteristics. The deviation between the realizations is small, indicating very little prediction uncertainty.

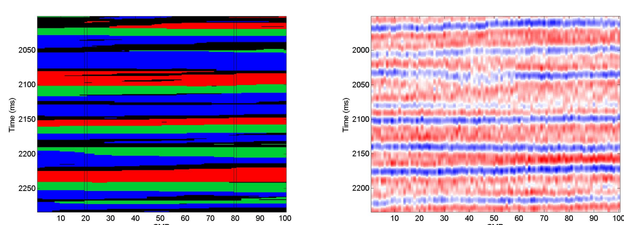


Figure 1: Reference LF characteristics $\pi$ with two well profiles marked, with gas-saturated sandstone (red), oil-saturated sandstone (green), brine-saturated sandstone (blue) and shale (black); and synthetic seismic data $d$ stacked from the angles $\theta = (0, 10, 20, 30, 40)$ °.

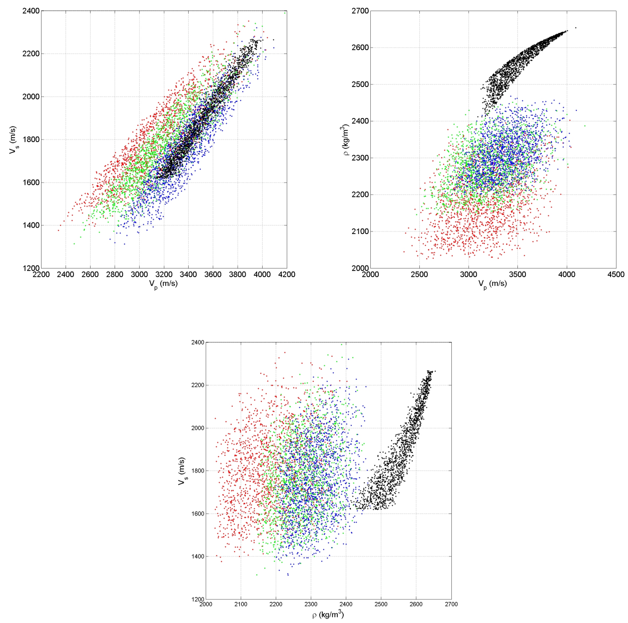


Figure 2: Elastic variables $m$ represented by P-wave velocity $(V_p)$, S-wave velocity $(V_s)$ and density $(\rho)$ given gas-saturated sandstone (red), oil-saturated sandstone (green), brine-saturated sandstone (blue) and shale (black).

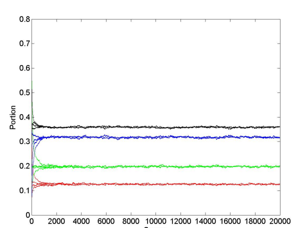


Figure 3: Convergence plot monitoring the portion of the LF classes after each sweep of the simulation algorithm with gas-saturated sandstone (red), oil-saturated sandstone (green), brine-saturated sandstone (blue) and shale (black)

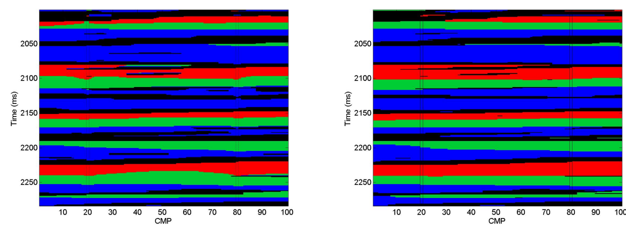


Figure 4: Locationwise most probable LF characteristics prediction $\hat{\pi}$; and reference LF characteristics $\pi$.

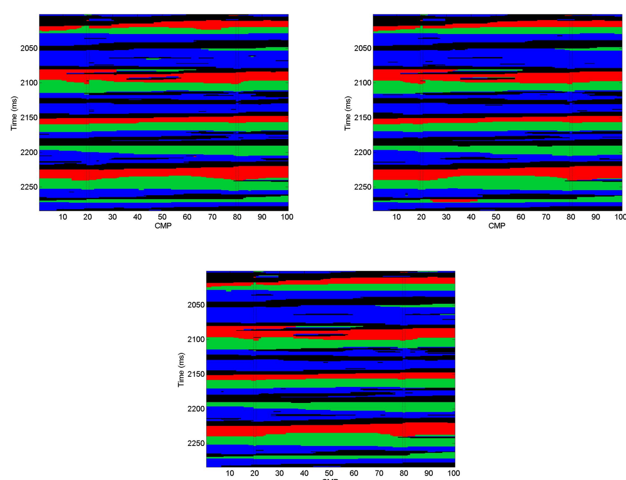


Figure 5: Independent realizations of LF characteristics from approximate posterior pdf $\tilde{p}(\pi|d)$.

## CONCLUSIONS

The LF inversion is based on a profile Markov random field prior model and an approximate likelihood model. The resulting approximate posterior model is explored using a block Gibbs sampling algorithm where one dimension is assessed using the recursive upward-downward algorithm. The inversion is evaluated on a synthetic case.

The prediction appears reliable. The LF characteristics have strong lateral and vertical continuity, and thin layers of shale are identified due to spatial couplings in the prior model. The model formulation makes it possible to include wells, such that the well information becomes an integral part of the solution.

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## REFERENCES

Besag, J (1974). S*patial interactions and the statistical analysis of lattice systems (with discussion)*. In Journal of the Royal Statistical Society, Series B (Methodological), vol. 36, no. 2, pp. 192–236.

Buland, A and Omre, H (2003). *Bayesian linearized AVO inversion*. In Geophysics, vol. 68, no. 1, pp. 185–198.

Larsen, AL, Ulvmoen, M, Omre, H and Buland, A (2006). *Bayesian lithology/fluid prediction and simulation on the basis of a Markov-chain prior model*. In Geophysics, vol. 71, no. 5, pp. R69–R78.